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5.   Entropy statistics as a framework to analyse technological evolution

Koen Frenken and Alessandro Nuvolari*

1.   INTRODUCTION

Many scholars have suggested that important similarities exist between technological development and biological evolution and that, for this reason, evolutionary models can provide us with fairly adequate representations of technical change (Nelson and Winter 1982, Basalla 1988, Mokyr 1990). However, as has been repeatedly pointed out by those who endorse the adoption of an evolutionary approach, there are also substantive differences between biological evolution and technological evolution (Freeman 1991, Nelson 1995). Therefore, evolutionary models should always be employed with caution, taking into account the specificities of the processes of mutation and selection under study.

The issue we are considering here concerns evolutionary processes of a special kind, namely the way complex entities evolve through processes of mutation and selection. Recent evolutionary theorizing in biology and artificial intelligence has stressed that complex entities evolve in ways that are different from non-complex ones in important respects. This claim also has significant implications for models of technological evolution, as a technological artefact is a complex evolving entity *par excellence* (Rosenberg 1976).

Following Simon's (1969 [1996]) work on the design of artificial systems, we describe a technological artefact as a man-made system constituted by interconnected components that are intended to collectively perform a number of functions. The complexity of an artefact is due to the interdependencies between components, which causes only some combinations of elements to work well together, in the sense that these combinations are capable of achieving satisfactory levels of performance. In Simon's view, a good deal of what we call innovative activities consists of trying to improve the general performance of the artefact by finding out progressively better configurations of its constituting elements.[1]

Until recently, however, formal treatments of system interdependencies

for the understanding of technological innovation have been scarce. This has changed with the introduction of 'complexity' models from natural sciences in the realm of (evolutionary) economics. In this respect, Kauffman's (1993) NK model of evolutionary biology has proven extremely promising and has already been adopted in a large number of contributions in the innovation and organization literature.[2] The NK model represents the design process of a complex technological artefact as a trial-and-error process that is bound to end up in a local optimum. Although the NK model has received considerable attention, much less effort has so far been put into empirical applications.[3] In this chapter, we set out a framework based on entropy statistics, which allows a relatively straightforward application of the NK model to empirical studies of technological change.

We consider the examples of the early development of the steam engine (1760–1800), the development of the aircraft (1913–84), and the development of the helicopter (1940–83) to illustrate the way in which the NK model can be employed in empirical studies of technical change by means of entropy statistics. As we will see, the interpretative accounts that we were able to produce using the NK model in combination with the entropy methodology emend the received histories of the technologies we are examining in this chapter in important respects. This suggests that other historical studies of technology could indeed benefit greatly from the adoption of the type of approach we propose in this study.

The remainder of the chapter is organized as follows. Section 2 contains an exposition of Kauffman's (1993) NK model and a number of generalizations since developed. Section 3 presents our entropy methodology in detail. Section 4 applies the entropy framework to data on steam engines, aircraft, and helicopters, and discusses the results in the light of received histories of these three technologies. Section 5 draws conclusions.

2.   TECHNOLOGICAL DEVELOPMENT AS A SEARCH PROCESS ON RUGGED LANDSCAPES

Many scholars have recognized that interdependencies between components in technological artefacts are the prime source of design complexity (Simon 1969 [1996], Rosenberg 1976, Sahal 1985, Vincenti 1990, Ziman 2000). The existence of interdependencies between components implies that the functioning of a system cannot be fully understood from the functioning of its individual components. Depending on the precise combination of the components that make up a system, a component will function in a different way. And, each time one manages to improve the functioning of

one component, new problems can arise in other components accordingly requiring redesign. In this context, Rosenberg (1976) introduced the concept of 'technical imbalances' between components that trigger sequences of problems and solutions over time.[4]

The existence of system interdependencies is what we understand to be the nature of complexity in the development of new technological designs. In this perspective, the design task essentially consists of the combinatorial problem of assembling the right set of components in a functioning system. The space of all the possible combinations between all the possible configurations of all the components of a system is called the 'design space' of a technology (Bradshaw 1992). Assume that a technology can be described by N components, or more generally, dimensions ($i = 1,...,N$). Along each dimension i there exist A_i possible states or configurations, called 'alleles', which can be coded as '0', '1', and so on. Each possible design can then be written as a string of alleles $s_1 s_2 \ldots s_N$ and is part of N-dimensional design space S, for which it holds that:[5]

$$s \in S; \; s = s_1 s_2 \ldots s_N; \; s_i \in \{0, 1, \ldots, A_i - 1\}. \qquad (5.1)^6$$

The combinatorial nature of a design space implies that the size of a design space increases exponentially for linear increases in N. The size of the design space S is given by the product of the number of alleles along each dimension:

$$S = \prod_{i=1}^{N} A_i \qquad (5.2)$$

In the case of binary strings (that is, when all dimensions contain only two alleles '0' and '1'), the size of design space equals $S = 2^N$, meaning that the number of possible designs doubles for each dimension added. As technological artefacts are typically made up of many dimensions and many alleles per dimension, they have enormous design spaces. Exploring the whole design space would obviously be very expensive. Instead designers will usually apply search rules that allow them to economize by examining only subsets of the design space. Thus only a small part of the design space will in effect be searched, and an even smaller part of the design space will be commercialized on product markets.

2.1   The NK Model

Kauffman and Levin (1987) and Kauffman (1993) developed the NK model to examine the properties of evolving complex systems with varying degrees of complexity. Complexity stems from interdependencies between

	$i = 1$	$i = 2$	$i = 3$
w_1	x	x	–
w_2	x	x	–
w_3	x	–	x

Figure 5.1   Example of an architecture of an N $=3$ *system with* K $=1$

the constituting dimensions of a system, such as genes in biological organisms and components in technological artefacts. The interdependencies between dimensions in a complex system are called 'epistatic relations'. An epistatic relation between components implies that when a component mutates, the mutation affects not only the functioning of the component itself but also the functioning of all the components that are 'epistatically related' to it. The ensemble of epistatic relations in a technological system is called a technology's architecture (Henderson and Clark 1990).[7]

The NK model is restricted to particular types of system architectures that can be expressed by a single parameter K, which stands for the number of other components that affect the functioning of each component. For example, the class of systems for which $K = 1$ holds refers to systems with an architecture in which the functionality of each component depends on the choice of allele of the component itself and on the choice of the allele of one other component. The K parameter can be considered an indicator of a system's complexity, with $K = 0$ being the least complex and $K = N - 1$ the most complex architecture. When $K = 0$ each technical dimension is independent of any other dimensions. Optimization can then proceed by optimizing each individual dimension separately, which will lead automatically to the global optimum. For increasing values of K it will become increasingly hard to optimize the system design globally, as interdependencies exist between dimensions. The number of local optima in which one can end up increases with the value of K.[8]

Consider, as an explanatory example, a system for which $N = 3$ and $K = 1$ hold, with an architecture as specified in Figure 5.1. Mutations in components in the columns affect the functioning of the component in the row as indicated by 'x'. The symbol '–' denotes that there is no epistatic relation between the component in the row and the component in the column. The architecture in Figure 5.1 specifies the following epistatic relations between the three components in the system. The functioning of the first component w_1 changes when the first component itself or the second component is mutated. The functioning of the second component w_2 changes when the
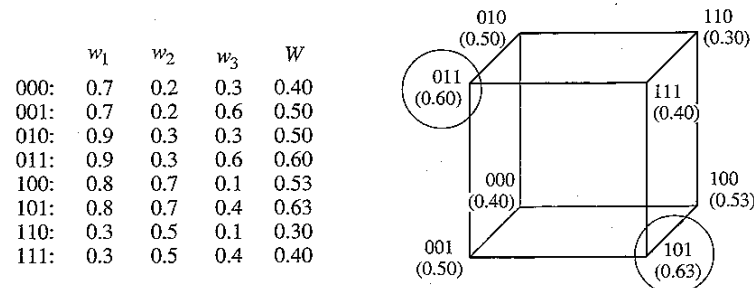
	w_1	w_2	w_3	W
000:	0.7	0.2	0.3	0.40
001:	0.7	0.2	0.6	0.50
010:	0.9	0.3	0.3	0.50
011:	0.9	0.3	0.6	0.60
100:	0.8	0.7	0.1	0.53
101:	0.8	0.7	0.4	0.63
110:	0.3	0.5	0.1	0.30
111:	0.3	0.5	0.4	0.40

Figure 5.2   Simulation of a fitness landscape of an N $=3$ *system with* K $=1$

second component itself or the first component is mutated. And the functioning of the third component w_3 changes when the third component itself or the first component is mutated.

Following Kauffman (1993), we construct a fitness landscape by drawing randomly the value of the fitness w_i of component i from the uniform distribution between 0 and 1. A random value is drawn for w_i each time component i is itself mutated and each time another component that epistatically affects component i is mutated. System fitness W is derived as the mean value of the fitness values of all components:

$$W(s) = \frac{1}{N} \cdot \sum_{i=1}^{N} w_i(s_i) \tag{5.3}$$

A simulation of a fitness landscape is given in Figure 5.2. The circled strings are local optima or 'peaks' on a 'rugged fitness landscape'. For these local optima it holds that all neighbouring strings, that is, the strings that can be reached by a mutation in one component, have a lower fitness W. In the simulation in Figure 5.2, this property holds for strings 011 and 101 as their system fitness values $W(011)$ and $W(101)$ exceed the values of their neighbouring strings. Local optima reflect complementary alleles as the collective fitness exceeds the value of neighbouring strings.

Using the concepts of design space and fitness landscape, the design process can be modelled as a local search process based on trial and error. Local search proceeds by means of a mutation in one, randomly chosen, dimension (a *trial*). A mutation means that a designer moves to a neighbouring string in the design space. The newly found string is accepted when system fitness W increases, while it is rejected when system fitness decreases

(error). Acceptance of a mutation implies that search continues from the newly found string, and rejection implies that search continues from the previous string. In this way, a designer can search for improvements in an incremental way until a local optimum is found that can no longer be improved by means of a mutation in one dimension.[9] Trial-and-error search can thus be considered as an 'adaptive walk' over a fitness landscape towards a local optimum, and the search will only halt when a local optimum is reached. Following the metaphor of the fitness landscape, this type of search in complex technological systems can be considered a process of 'hill-climbing'.

It should be stressed that we used the relatively simple case of $N=3$ in the example above for explanatory purposes. In real-world R&D activities, the number of design dimensions N is generally much larger. Consequently, local search takes place in much larger design spaces containing many more local optima for the same value of K. The probability of ending up in a local optimum is correspondingly much higher.

An important property of the NK model holds that the number of local optima in a fitness landscape is a function of the complexity K of a system's architecture. When complexity is absent $(K=0)$, the fitness values of each dimension are not affected by mutations in other dimensions. Therefore, the global optimum of a system of $K=0$ can always be found by local search through trial and error as described above. Put another way, fitness landscapes of $K=0$ systems always contain only one optimum (which is by definition the global optimum). For systems with a positive K value, the fitness values of dimensions are affected by mutations in dimensions that are epistatically related. As a result, the fitness landscape will generally contain multiple local optima. Kauffman (1993) has shown that the expected number of local optima increases for increases in K. This means that it becomes increasingly hard to find the global optimum for systems with higher complexity.

A second property of the NK model holds that the fitness of local optima decreases for increases in K. One can understand this outcome as reflecting the detrimental effects of a higher number of conflicting constraints between components. The higher K, the more difficult it becomes to improve the fitness of one component without lowering the fitness of other components. Consequently, the system fitness of local optima is generally quite low. Furthermore, the variance of fitness value of local optima also decreases for increases in K, which means that the differences in fitness of local optima become smaller for systems with higher complexity. In the context of competing technological designs, this result suggests that the higher a technology's complexity, and the smaller the performance differences between locally optimal designs, the more persistent design variety will be.[10]

2.2   Generalizations of the NK Model

The NK model can be generalized in a number of respects to represent a wider range of phenomena. The first generalization concerns the representation of the relation between a system's 'genotype' (the set of design dimensions) in relation to the 'phenotype' (the set of functions a system performs). The NK model is based on the idea that each component of the system performs an 'own' sub-function within the system with regard to the attainment of one overall function on which external selection operates (Kauffman 1993, p. 37). Each component i is conceived to have a particular fitness value w_i that reflects its functional contribution to the system as a whole. The fitness of the system as a whole is derived as the average of the fitness of individual components.

Altenberg (1994, 1995, 1997) describes a generalized (biological) model of complex systems that contains N dimensions $(i=1,...,N)$ and F functions $(f=1,...,F)$ and for which it holds that N does not necessarily equal F. In biological systems, for which the original and generalized NK models were both initially conceived, an organism's N genes are the system's components and an organism's F traits are the system's functions on which natural selection operates. The string of genes constitutes an organism's genotype and the set of traits constitutes an organism's phenotype. The genotype of an organism is the level at which mutations take place, which are transmitted to offspring. The phenotype is the level at which natural selection operates in terms of its relative fitness.

Analogously, a technological artefact can be described in terms of its N components and the F functions it performs. The string of alleles describes the 'genotype' of a technological system, and the list of functions describes the 'phenotype' of this system. Typical functions of technological artefacts include cost-related criteria (fuel-efficiency, maintenance cost, and so on) and performance-related criteria (power, speed, weight, safety and the like).[11]

In Altenberg's generalized NK model, the architecture of a complex system is represented by a 'genotype–phenotype matrix' of size $F \times N$ with:

$$M = [m_{fi}], f = 1, \ldots, F, i = 1, \ldots, N \qquad (5.4)$$

As in the NK model, an epistatic relation is represented by 'x' when function f is affected by component n and by '–' when function f is not affected by the component n. An example of a matrix for $N=3$ and $F=2$ is given in Figure 5.3.

The way in which fitness landscapes are constructed for generalized genotype–phenotype matrices follows the same logic as the original NK model

	$i = 1$	$i = 2$	$i = 3$
w_1	x	x	—
w_2	—	x	x

Figure 5.3   Example of a generalized genotype–phenotype matrix

discussed in the previous section. For each component that is mutated, all functions that are affected by this component are assigned a new, randomly drawn fitness value w_f from the uniform distribution between 0.0 and 1.0. Total fitness W is again derived as the mean of the fitness values of all functions:

$$W(s) = \frac{1}{F} \cdot \sum_{f=1}^{F} w_f(s) \qquad (5.5)$$

A simulation of the fitness landscape example of the genotype–phenotype matrix of Figure 5.3 is given in Figure 5.4 for all possible combinations between two alleles of three components. Local optima are again circled, reflecting the combinations in which component technologies are complementary.

The meaning of the concepts of fitness landscape and local optima remains entirely the same in Altenberg's (1994, 1995, 1997) generalized NK model. Moreover, the properties of the NK model discussed above, which relate the number of local optima, the fitness of local optima, and the variance in fitness of local optima to complexity K, remain intact. The main difference compared to the original NK model is that in the generalized model the number of dimensions N is not necessarily equal to the number of functions F. Altenberg's (1997) model can therefore be considered as an important generalization of the original NK model of complex systems by Kauffman (1993).[12]

A second generalization of the NK model can be introduced by specifying a more general fitness function that translates the fitness levels of individual functions w_f into one overall assessment value W. The fitness function in Equation (5.5) specified that each function is weighted equally. As an empirical specification of fitness (performance) of a technology, this equation obviously does not account for the general case in which users may apply different weights to the various functions of the artefact. Allowing for different values of weights for each function, we get:

$$W(s) = \sum_{f=1}^{F} \beta_f \cdot w_f(s) \qquad (5.6)$$

$$\sum_{f=1}^{F} \beta_f = 1, \; \beta_f > 0 \qquad (5.7)^{[13]}$$

A selection environment can then be defined by the set of weights $\{\beta_1, \beta_2, \ldots, \beta_F\}$ that is applied by users of the technology. The concept of a fitness landscape does not change when total fitness is computed as a weighted sum instead of as the average of the fitness values of functions. However, the values of total fitness of each design $W(s)$ will be different depending on the values of the weights that are applied.

A final generalization can be introduced by allowing for heterogeneity among users. So far, we have implicitly assumed that each user of a particular design applies the same set weights and thus assigns the same fitness value $W(s)$ to a design. However, depending on the specific use of the design, different users may well apply different weights, and thus assign different fitness values to one and the same design (Lancaster 1966, 1979, Saviotti and Metcalfe 1984, Saviotti 1996).[14] In this case of heterogeneous demand, different users have different valuations of the same technological design, as they weight the levels of functions differently. As Lancaster (1979, p. 17) expressed it:

> Differences in individual reactions to the same good are seen as expressing different preferences with respect to the collection of characteristics possessed by that good and not different perceptions as to the properties of the good.

The weights assigned to functions as specified above $\{\beta_1, \beta_2, \ldots, \beta_F\}$ reflect one homogeneous user group. When there is more than one user group, we

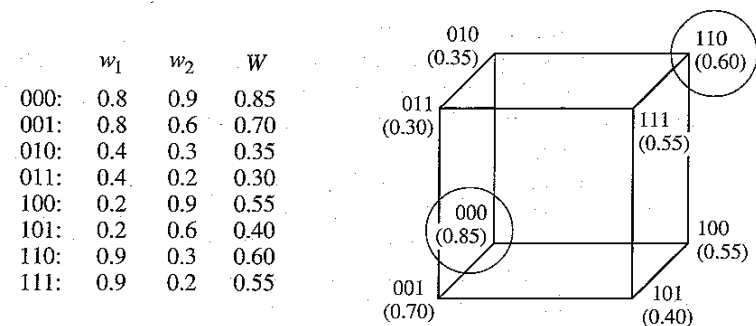|	w_1	w_2	W
000:	0.8	0.9	0.85
001:	0.8	0.6	0.70
010:	0.4	0.3	0.35
011:	0.4	0.2	0.30
100:	0.2	0.9	0.55
101:	0.2	0.6	0.40
110:	0.9	0.3	0.60
111:	0.9	0.2	0.55

Figure 5.4   Simulation of fitness landscape of the matrix in Figure 5.3

can characterize each different user group by a different set of weights. For a G number of user groups g ($g = 1,...,G$), we have G sets of weights. For each user group, the fitness W_g of a design is given by:

$$W_g(s) = \sum_{f=1}^{F} \beta_{fg} \cdot w_f(s) \qquad (5.8)$$

$$\sum_{f=1}^{F} \beta_{fg} = 1, \; \beta_{fg} \geq 0 \qquad (5.9)^{[15]}$$

This specification of the selection environment includes the specification given above for a homogeneous selection environment as the special case in which $G = 1$.

When heterogeneity in preferences is more dispersed, it is less likely that one design is optimal for all user groups. In that case, product differentiation is expected to occur. In the extreme case, given a sufficiently large design space, a different design may be found for each different user group. When a user group exists for which no design is yet optimized, and this is known to designers, this in itself can spur the search for innovations in particular components in order to find a new design capable of fitting their particular demand (induced innovation).

2.3.   IMPLICATIONS

Regarding the patterns of technological evolution one can expect to emerge, a number of implications follow from the previous discussion of the NK model and its generalizations. To summarize the implications, one can distinguish between technologies that are subject to homogeneous demand and technologies that are subject to heterogeneous demand:

- When demand is homogeneous ($G = 1$) and complexity is absent ($K = 0$) there exists only one local optimum, which can be found by local trial and error. When demand is homogeneous ($G = 1$) and system complexity is present ($K > 0$), the expected number of local optima becomes a function of the complexity parameter K. Thus, even when demand is homogeneous, technological variety is expected to emerge as different designers will come up with different, locally optimal solutions. What is more, the variety in technological designs can be quite persistent for highly complex architectures, as the variance in the fitness values of local optima has been shown to decrease as a function of K (Kauffman 1993).
- When demand is heterogeneous ($G > 1$) and complexity is absent

($K = 0$), there is only one global optimum, which is the same for each user group, since each function can be optimized independently from other functions. The existence of heterogeneous demand is not a sufficient condition for design variety to emerge. When demand is heterogeneous ($G > 1$) and complexity is present ($K > 0$), design variety is expected to emerge for two reasons. First, as in the case of homogeneous demand, trial and error may lead different designers to come up with different local optima. Second, the heterogeneity in preferences may render different designs to be globally optimal for different user groups. Note that in the case of heterogeneity in preferences, the design variety is expected to be even more persistent than in the case of homogeneity in preferences where variety may slowly disappear as sub-optimal design lose ground due to small differences in fitness.

Note that design variety that is expected to emerge following the generalised NK-model is always limited by the extent to which scale economies and network externalities are realized in the production and use of a single design (Arthur 1989). When one design s is produced and used in much higher numbers than alternative designs, lower price and higher willingness-to-pay may attract users who previously preferred an alternative design. However, a greater degree of heterogeneity of preferences as expressed by the different weights users assign to the various functions, will in turn render it less likely that one design will attract all users. Moreover, radical innovation may at all times lead to the introduction of complete new designs attracting new users or users who previously adopted another design.

Patterns of technological evolution thus depend crucially on the complexity of a technology's architecture, the number of functions that can be distinguished, and the degree of heterogeneity of demand. In the three cases we will discuss below (steam engines, aircraft, and helicopters), the complexity and number of selection criteria is generally estimated to be quite high (Rosenberg 1982). Moreover, all three technologies have been used in a wide range of user contexts. In our view, one can expect an empirical analysis to show technological variety to emerge in the course of their evolution.

3.   ENTROPY STATISTICS

In the previous section we proposed a formalization of artefact complexity and discussed its implications for the patterns of technological evolution

that are expected to emerge. A straightforward way to analyse empirical data on artefact designs in terms of the ruggedness-of-fitness landscapes is to apply entropy statistics. Entropy statistics can be computed using frequency distributions of technological designs coded in the N-dimensional design space and they allow one to map both the degree of technological variety (by means of entropy indices), and the nature of technological variety (by means of mutual information indices). In this way, evolutionary trends in the development of a technology can be consistently outlined.

The entropy index refers to the degree of randomness in the choice of technological designs as reflected by the skewness of a distribution. A skewed distribution reflects a situation in which designers hardly differ in their choice of design, while a flat distribution reflects a situation in which designers have come up with very many different designs. As such, entropy can be used as an indicator of technological standardization and to what extent a dominant design can be said to have emerged (Frenken et al. 1999b). The more skewed a distribution, the lower the entropy (randomness) of a distribution.

To understand to what extent the variety indicated by entropy can indeed be said to reflect local optima on a rugged-fitness landscape, a second indicator called mutual information is introduced (Frenken 2000, 2001). Mutual information indicates the extent to which particular alleles along different dimensions co-occur in the technological designs offered on the market. Statistically, mutual information thus indicates the degree of dependence between different design dimensions. The existence of local optima would imply that particular alleles along one dimension typically co-occur often with particular alleles along other dimensions, which would result in a high value of mutual information (dependence). Following the metaphor of a fitness landscape, high mutual information indicates that designers occupy more than one peak. When alleles along different dimensions are more or less randomly combined, mutual information is low (independence). Designers are more or less randomly spread out over the fitness landscape without clustering around specific peaks.

3.1 Entropy

The entropy concept was developed in late nineteenth-century thermodynamics to describe randomly moving particles (Prigogine and Stengers 1984). When many particles are moving randomly through a state space, like particles of a gas in a box, the resulting distribution of all particles is completely flat. The flat distribution follows from the fact that at all times each particle has an equal probability of being present in any area in the box. The flat distribution is characterized by maximum entropy (random-

ness). When particles behave in a non-random way, some areas in the box will be filled with more particles than other areas, and the resulting distribution is skewed. In that case, the entropy of the distribution is lower compared to the case in which all particles move randomly. In the extreme case when all particles cluster in one area of the box, entropy is lowest.

Entropy is thus a macroscopic measure at the level of a distribution that indicates the degree of randomness in the microdynamics underlying a frequency distribution. As such, entropy can also be used as a variety measure of frequency distributions of technological designs. Following Saviotti (1996), we refer to a distribution of technological designs as the 'product population'. Maximum entropy corresponds to the case in which all designs occur at the same frequency. Such a completely flat distribution would occur when designers move around randomly in state space, which has been called here the 'design space'. In that case, designers pick randomly the various alleles of each component. In this hypothetical case, any product design has an equal probability of occurrence, and the product population would be characterized by even frequencies of all designs. This hypothetical situation refers to a situation in which designers do not learn about the functional properties of different designs, and simply choose the alleles configuration at random (analogous to the randomly moving particles in a box, explained above). A skewed distribution occurs when some designs dominate the product population. In that case, the frequency of some designs is high, while the frequency of most designs is low or zero. In this case, designers have not chosen a design at random, but have somehow learned which designs are most demanded, for example, by applying a local search strategy of hill-climbing. In the extreme case in which all designers choose to offer one and the same design on the market, entropy will be minimum.

The entropy measure thus indicates the degree of design variety in a product population. To describe a product population as a frequency distribution of designs, let each design be coded again as a string of N alleles ($i = 1, \ldots, N$). Each of the N dimensions is labelled here as X_i, with each dimension containing A_i alleles again coded as '0', '1', and so on. The relative frequency of design s in the product population is denoted as p_s. The entropy value of an N-dimensional distribution is then given by (Theil 1967, 1972, Langton 1990):

$$H(X_1, \ldots, X_N) = -\sum_{s_1=0}^{A_1-1} \cdots \sum_{s_N=0}^{A_N-1} p_s \cdot \ln p_s. \qquad (5.10)^{[16,17]}$$

Entropy is zero when all products present in the population are designed according to one and the same design. This design would have a frequency of one in the product population, which implies that the entropy of the product population equals:

$$H_{\min} = -1 \cdot \ln(1) = 0.$$

Entropy is positive otherwise. The larger the entropy value, the larger the design variety in the product population. The maximum entropy is limited by the size of design space S. When all S possible combinations of alleles have an equal frequency, we obtain a uniform distribution in which each design has frequency $p_s = 1/S$. The entropy of this distribution equals:

$$H_{\max} = -S \cdot \left(\left(\frac{1}{S} \right) \cdot \ln \left(\frac{1}{S} \right) \right) = -\ln \left(\frac{1}{S} \right) = \ln(S).$$

This value is the maximum possible entropy value for a distribution of product designs with a design space of S possible designs. It implies that for larger values of S, maximum entropy increases, with the marginal increase of maximum entropy decreasing. This property reflects that each new entity added contributes to variety, but decreasingly so.

Similarly, the design variety along one dimension i can be computed. The one-dimensional or marginal entropy indicates the variety in a product population with respect to one design dimension only, and is given for each dimension by:

$$H(X_i) = -\sum_{s_i=0}^{A_i-1} p_{s_i} \cdot \ln p_{s_i}. \tag{5.11}$$

As we will see, the one-dimensional entropy formula can be used to compute the mutual information index, which is equal to the difference between the sum of one-dimensional entropy values and the N-dimensional entropy value.

3.2 Mutual Information

In information theory, the measure that indicates the degree of dependence (co-occurrence of alleles) in a frequency distribution is the measure of mutual information T. Mutual information is given by (Theil 1967, 1972, Langton, 1990):

$$T(X_1, \ldots, X_N) = \sum_{s_1=0}^{A_1-1} \cdots \sum_{s_N=0}^{A_N-1} p_s \cdot \ln \frac{p_s}{\prod_{i=1}^{N} p_{s_i}}. \tag{5.12}$$

The mutual information value T indicates the extent in which alleles along different dimensions are co-occurring in the distribution of designs. The mutual information value equals zero when there is no dependence between any of the dimensions. In that case, the joint frequency of alleles of components p_s corresponds exactly to the frequency that could be expected

from the product of the marginal frequencies $\Pi_{i=1}^{N} p_{s_i}$. When the product of marginal frequencies does not correspond to the joint frequency, there is dependence between dimensions. Mutual information is thus derived by the weighted sum of dependence values for each design. It can be proven that the weighted sum of dependence values is non-negative for any frequency distribution; that is, $T \geq 0$ (Theil 1972). The greater the difference between the joint frequency and the product of marginal frequencies, the higher the value of the mutual information, and the more alleles along particular dimensions co-occur in 'design families'.

The mutual information measure is directly related to the concept of entropy as mutual information can be derived from the multi-dimensional and marginal entropy values. In the general case of an N-dimensional distribution $(N > 1)$ the mutual information equals the sum of marginal entropy values minus the N-dimensional entropy value (Theil and Fiebig 1984, p. 12):

$$T(X_1, \ldots, X_N) = \left(\sum_{i=1}^{N} H(X_i) \right) - H(X_1, \ldots, X_N). \tag{5.13}$$

From this equation, it can be derived that the mutual information equals zero if entropy equals zero, and that mutual information equals zero if entropy is maximum (see Appendix).

Similarly, one can compute the mutual information between each pair of dimensions to indicate dependence between two dimensions:

$$T(X_i, X_j) = H(X_i) + H(X_j) - H(X_i, X_j) \quad i \neq j; i = 1, \ldots, N; j = 1, \ldots, N.$$

The two-dimensional mutual information values indicate the dependence between a pair of dimensions and are thus informative with regard to the importance of epistatic relations among the pair of dimension in question. A high mutual information between two dimensions suggests that an important epistatic relation exists between the two dimensions, since designers predominantly offer alleles in particular opposite combinations (for example, either combination 00 or combination 11). Dependence reflects dominant complementarities between two dimensions as particular alleles along the one dimensions often co-occur with particular alleles along the other dimension and irrespective of alleles in yet other dimensions.

3.3 Entropy and Mutual Information as Indicators of Evolution

To explain the connection between entropy and mutual information indicators and the exploration of rugged-fitness landscapes, one should keep in

Table 5.1   Three examples of distribution for binary strings of N = 3

Distribution	p_{000}	p_{001}	p_{010}	p_{011}	p_{100}	p_{101}	p_{110}	p_{111}
Case 1	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
Case 2	0.250	0.000	0.000	0.250	0.000	0.250	0.250	0.000
Case 3	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.500

Entropy	$H(X_1,X_2,X_3)$	$H(X_1,X_2)$	$H(X_1,X_3)$	$H(X_2,X_3)$	$H(X_1)$	$H(X_2)$	$H(X_3)$
Case 1	ln 8	ln 4	ln 4	ln 4	ln 2	ln 2	ln 2
Case 2	ln 4	ln 4	ln 4	ln 4	ln 2	ln 2	ln 2
Case 3	ln 2	ln 2	ln 2	ln 2	ln 2	ln 2	ln 2

Mutual information	$T(X_1, X_2, X_3)$	$T(X_1, X_2)$	$T(X_1, X_3)$	$T(X_2, X_3)$
Case 1	0	0	0	0
Case 2	ln 2	0	0	0
Case 3	ln 4	ln 2	ln 2	ln 2

mind the relationship between entropy and mutual information. Recall Formula (5.13), which expresses mutual information as the sum of marginal entropy values minus multi-dimensional entropy, which can be rewritten as:

$$\left(\sum_{i=1}^{N} H(X_i) \right) = T(X_1, \dots, X_N) + H(X_1, \dots, X_N)$$

From this formula, it can readily be seen that, given a value for the sum of marginal entropy (ΣH_i), mutual information can increase only at the expense of (total) entropy, and vice versa. This relationship is illustrated in Table 5.1, in which three different frequency distributions of designs are listed (for $N = 3$). In all three cases, the sum of marginal entropy values is the same $(\Sigma H_i = 3 \cdot \ln(2) = \ln(8))$, because in all three cases the two alleles along each dimension occur at the same frequencies. However, the three-dimensional entropy and mutual information values differ for each distribution. Case 1 corresponds to a uniform distribution with maximum entropy and zero mutual information. Case 2 shows that a multi-modal distribution with four designs has positive and equal frequencies. Three-dimensional entropy equals ln(4), while three-dimensional mutual information is only ln(2). Finally, case 3 shows a bi-modal, 50–50 distribution in which two opposite designs are present in the product population (000 and 111). In this case, three-dimensional entropy equals only ln(2), while three-dimensional mutual information adds up to ln(4). The latter case is characterized by such

high mutual information because knowledge of one allele along one dimension of a design would allow one to perfectly predict the alleles along the two other dimensions.

When entropy and mutual information are applied to the frequency distributions of consecutive years of technological evolution, a very different picture may emerge. In that case, the value of ΣH_i in a particular year will differ from the value of ΣH_i in other years. Over time, the value of ΣH_i may increase or decrease, or show no trends. An increasing trend would indicate a growing variety in alleles used along each design dimension. Following the formula, such an increase in the value ΣH_i implies that entropy and mutual information can both increase at the same time. In that case, we have a pattern of increasing design variety as indicated by the rise in $H(X_1 \dots X_N)$ *and* of increasing differentiation of designs in families as indicated by the rise in $T(X_1 \dots X_N)$. Such a process indicates the progressive development of a growing number of design families akin to 'speciation' in biology (Saviotti 1996, Levinthal 1998). The reverse pattern can also take place. When ΣH_n is falling, entropy and mutual information may decrease at the same time (for example when a product family totally disappears).

The evolutionary development of a complex technology, following the generalized NK model as discussed earlier, is expected to be characterized by both an increasing degree of variety (entropy) and an increasing degree of differentiation (mutual information). Such a development process can be understood from the multi-dimensional and complex nature of technological artefacts and the existence of heterogeneous demand.

4. APPLICATIONS

We will test our thesis of growing design variety and differentiation into design families using data on early steam engines (1760–1800), aircraft (1913–84) and helicopters (1940–83). For each technology we will first provide a short summary of the 'standard' historical account of its development, then present the data and results, and finally discuss what new insights can be derived from the analysis.

4.1   Steam Engines

4.1.1   Early steam-engine history[18]
Historians of technology have described the early development of steam-power technology as a 'linear' succession of technological breakthroughs. The main contours of what might be called the traditional

account[19] of early steam-engine development concern the design sequence of Savery–Newcomen–Watt–Trevithick that took place during the eighteenth century.

In the late seventeenth century mining activities begun to be severely hampered by flooding problems. Following the scientific investigations of Torricelli and Pascal, there were several attempts to use atmospheric pressure to lift water out of mines. The Savery engine can be considered as the first successful effort in this direction. The engine was developed during the period 1695–1702. In the Savery engine, steam was first admitted and then condensed inside a 'receiving' vessel by pouring cold water over its outside. Following steam condensation, atmospheric pressure drove water up into the vessel. The engine had two major shortcomings, which limited its practical utilization: restricted height of operation and high fuel consumption due to the need for recreating steam inside the vessel at each stroke.

The Newcomen engine, developed in 1712, resolved the problem of limited height of operation. The Newcomen engine consisted of a piston–cylinder arrangement connected with one hand of a working beam. Steam was admitted from the boiler into the cylinder by means of a valve. Then a cold of jet of water was sprayed into-the cylinder, condensing the steam. At this point, because of the creation of a partial vacuum, atmospheric pressure pushed the piston down, lifting the pump rod at the other end of the beam. The use of the piston–cylinder arrangement together with the beam made it possible to use the engine for effective mine drainage. Furthermore, the Newcomen engine was robust, highly reliable and based on a fairly simple working principle. The Newcomen engine, however, did not solve the problem of high fuel consumption. Neither did the engine design deliver smooth motion, preventing the use of this kind of engine in applications in which a smooth rotary motion was needed.[20]

James Watt in the 1770s and in the 1780s successfully tackled these two problems. In his engine, condensation was carried out in a separate vessel and not in the cylinder. This design implied that there was no longer the need to reheat the cylinder at each stroke, which greatly contributed to fuel-efficiency. After the invention of the separate condenser, Watt conceived a number of modifications to his engine in order to allow the effective transformation of reciprocating motion into rotary motion. Among the designs that were developed for rotary motion was the double-acting Watt engine, in which steam is admitted into the cylinder on both sides of the piston in an alternating manner. This resulted in a more powerful action, but also in a much more regular movement of the piston.

Finally, in the second half of the 1790s, Richard Trevithick developed the first high-pressure engine (Watt engines used steam at a little more than atmospheric pressure). This type of engine did not use the separate con-

denser, but discharged exhaust steam directly into the atmosphere. For this reason, they were called 'puffers'. The main advantage of this type of engine was their compactness and their cheaper cost of installation due to elimination of the condenser, the air pump and the beam.[21]

As is apparent from this narrative, such a historical depiction is akin to chronicling a sort of 'glorious march of invention', where most of the emphasis is put on the creative contributions of a succession of individual inventors (the line Savery–Newcomen–Watt–Trevithick). Each inventor tackled the shortcomings of the technological 'state of the art', devising improvements that made previous engine designs obsolete through a process of technological substitution. The question is whether this traditional picture also emerges from entropy analysis.

4.1.2 Early steam-engine data

The data we use are taken from an up-to-date version of the database collected by John Kanefsky.[22] The database contains a list of all steam engines (more precisely, those for which some historical evidence has been found) erected in Great Britain over the period 1700–1800. We have limited ourselves to the period 1760–1800, as the period before 1760 was entirely dominated by the Newcomen design and thus was characterized by absence of variety and differentiation.[23]

The database contains 1370 engines for the period 1760–1800. Each of these engines is coded as a string of seven alleles that describes the engine design as a point in a seven-dimensional design space. Dimensions and alleles are given in Table 5.2. The design dimensions have been constructed in such a way that each design could be coded as a unique string, thus covering the most relevant dimensions of early steam-engine technology. After having coded each engine in the database as a design string according to the classification of the design space in Table 5.2, we constructed yearly frequency distributions and computed the entropy and mutual information values.

Note that we have considered three-year moving averages of the yearly entropy and mutual information values in order to smooth short-term fluctuations and obtain a 'neater' pattern. The results in the figures are shown per year, where each year stands for the in-between year of a three-year period. The transformation of yearly values into three-year moving averages does not in any way affect our conclusions.

### 4.1.3 Results on steam engines

From the results, it immediately becomes clear that variety (entropy) and differentiation (mutual information) have both increased very rapidly from 1774 onwards when the Watt engine became a popular design next to the

Table 5.2   Design space of steam-engine technology (S = 192)

Steam engine	
Number of observations:	*1370*
Time span:	*1760–1800*
Area:	*Great Britain*
X_1 $A_1 = 2$	Pressure 0 low, 1 high
X_2 $A_2 = 2$	Condenser 0 yes, 1 no
X_3 $A_3 = 2$	Action 0 single acting, 1 double acting
X_4 $A_4 = 2$	Compounding 0 yes, 1 no
X_5 $A_5 = 3$	Motion 0 reciprocating, 1 rotary, 2 water returning
X_6 $A_6 = 2$	Top 0 open, 1 closed
X_7 $A_7 = 2$	Cylinder 0 single, 1 double

older Newcomen design (Figure 5.5). The rise in variety and differentiation levelled off around ten years later (more or less from 1785). What is also clear is that, as both entropy and mutual information have been rising, the sum of marginal entropy values must have risen also, following Formula (5.13). This shows that the technological evolution of the steam engine has been characterized by the introduction of new alleles in several dimensions accounting for the rise in the sum of marginal entropy values. The introduction of new alleles has been such that both the variety in designs and the degree of differentiation in design families have risen. Put another way, the design variety has been made possible by the development of new alleles that are combined in highly non-random ways.

Closer inspection of Figure 5.5 also shows that during the 1770s and early 1780s the rise of entropy precedes increases in mutual information. We understand this as probably being due to the fact that new combinations of alleles were tried first, leading to an increase of variety. However, some of these new combinations did not reach adequate levels of fitness, and so we see that, with a delay, mutual information 'catches up' with the
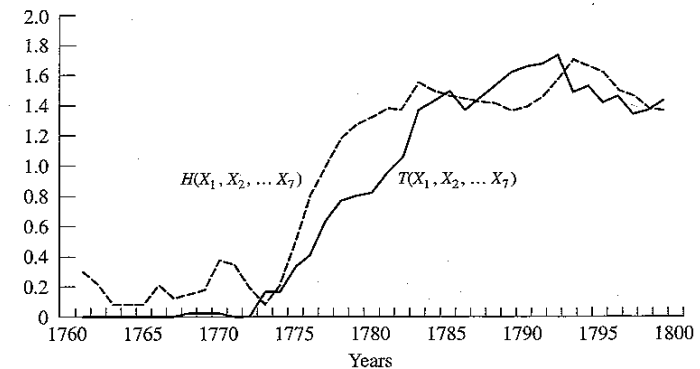
Figure 5.5   N-*dimensional entropy* (H) *and mutual information* (T) *for steam-engine designs*

entropy, which means that the product population is clustering around some specific points of the landscape. In other words, we have first a phase of exploration and discovery of new areas of the landscape, followed by concentration in some points that are likely to be local optima. The 'levelling-off phase' seems to suggest that from the late 1780s a stable pattern of differentiation finally emerged.

Results on two-dimensional mutual information values are depicted in Figure 5.6. The figure shows along which couples of dimensions differentiation has been most pronounced. Hence, these results are also informative about the nature of the technological interdependencies (epistatic relations) among the constituting elements of our design space. The highest mutual information values are reached by the pair $T(X_2, X_6)$, which reflect the interdependence between condensation and the closed-top cylinder. Separate condensation and the closed top cylinder are the two salient features distinguishing Watt type of engines (0100010) from the Newcomen atmospheric engine without condensation and open top (0000000). Importantly, the high values of $T(X_2, X_6)$ are not temporary but continue during the whole period considered. These results thus confirm the thesis of an emergence of a pattern of differentiation.

The other couples of dimensions with high mutual information values are $T(X_2, X_3)$, $T(X_2, X_5)$, $T(X_3, X_5)$, $T(X_3, X_6)$ and $T(X_5, X_6)$. What becomes clear from these results is that the high values are limited to four dimensions: X_2, X_3, X_5 and X_6 (respectively, with/without condenser, single/double action, reciprocating/rotary/water returning, and open/closed top). As explained above, dimensions X_2 and X_6 differentiate Newcomen and Watt
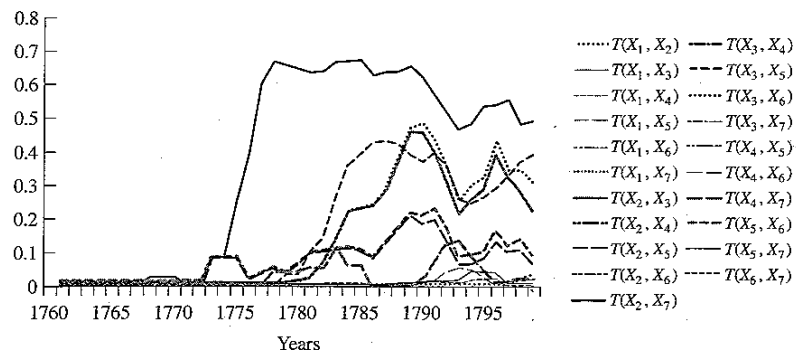
Figure 5.6   Two-dimensional mutual information (T) for steam-engine designs

engines. Dimensions X_3 and X_5 concern different types of solutions to deliver particular types of motion. Double action was a typical feature of Watt rotary engines (0110110), while Newcomen engines delivering rotary motion made use of either returning a stream over a waterwheel (0000200) or directly by alternatively using two cylinders (0000101).

4.1.4   Discussion

The pattern of growing variety and differentiation of early steam-engine technology suggests that newly developed designs did not simply substitute older designs, but enlarged total design variety. Our analysis shows that the 'linear' view of the early history of the steam engine is essentially untenable. Instead, technological evolution in this period is better characterized by progressive differentiation into distinct design families.[24] In terms of the NK model, the clustering of the product population around some specific designs can be seen as a reflection of designers occupying local optima in the fitness landscape.

The process of differentiation proceeded along four specific technical dimensions (with/without condenser, single/double action, reciprocating/rotary/water returning, and open/closed top). These dimensions may well be related to different user contexts, in particular to the cheapness of coal and to the desired properties of the rotary motion. The pattern of specialization we find contradicts received histories of early steam-engine evolution that point to a process of substitution between Watt engines and Newcomens. Although Watt's inventions are considered to have solved the main shortcomings of the Newcomen engine, it is mis-

leading to assume that they led to the substitution of Newcomen engines.

Regarding the superiority of Watt's fuel efficiency, one can understand the limited substitution of Newcomen engines by Watt engines, taking into account the higher costs of erection and maintenance of the Watt engine. In this respect, von Tunzelmann (1978) has argued that in areas where coal was cheap enough, the Newcomen engine had an important advantage due to its lower costs of installation and maintenance. Besides, whereas the Newcomen engine was well within the engineering capabilities of the time, the Watt engine imposed very compelling requirements on the degree of accuracy of the various components of the engine. This points to the existence of a fundamental trade-off concerning fuel-efficiency versus simplicity of construction and maintenance.[25]

Regarding the type of motion that Watt engines were capable of delivering, the significance of Watt's design modifications also requires further nuance. Although Watt's inventions for supplying rotary motion were highly celebrated (Dickinson and Jenkins 1927), they should not by any means be considered definitive, especially given the accuracy of workmanship of the time. We are aware of many cases of unsatisfactory performance of Watt rotary engines in textile mills.[26] This explains why Watt engines only partially substituted alternative designs that delivered rotary motion.

Interestingly enough, there was an attempt to develop a 'hybrid' engine combining the simplicity of Newcomen with the fuel-efficiency of Watt. This was the 'improved atmospheric engine' patented by Symington in 1787 (0100000). Unfortunately, we have scant information on this engine (especially on its actual fuel-efficiency compared to Watt). We know that about twenty of this type of engine were erected mainly in Scotland and that they generally proved rather successful.[27] Some historians of technology (Dickinson and Jenkins 1927) have dismissed Symington simply as a 'schemer' who tried to circumvent Watt's patent.[28] Our results instead suggest that his attempt to merge the two separate design trajectories of the Newcomen and Watt designs was genuinely aimed at solving a teething trade-off.

To summarize, the existence of various user contexts implied that engine designs be differentiated in order to provide adequate responses to the specific demands of the various user sectors. In our case, this determined a divergence of design trajectories, a process akin to speciation in biology. In a companion paper (Frenken and Nuvolari 2002), we study the pattern of specialization of different type of steam engines in the various user contexts in greater detail using data on the sector of application of engines.

4.2   Aircraft

4.2.1   Aircraft history

Both historians and economists have analysed the development of air-craft technology in considerable detail (for example Miller and Sawers 1968, Constant 1980, Bilstein 1996). Although these studies differ in their perspectives and methodologies, there is a general consensus on the main stages of aircraft development, which can be divided into four periods.

The early history of aircraft from the turn of the century to roughly 1930 is characterized by a large variety of designs and limited demand. A large number of new, small firms experimented with various designs and materials. This period is commonly considered an explorative stage in the industry characterized by a great deal of trial and error. During this period, series production remained limited, causing production costs and prices to be too high for mass consumption.

The second stage, covering the 1930s and early 1940s, has been marked as the period of technological convergence towards what has been termed a 'dominant design' (Abernathy and Utterback 1978). The Douglas DC3 developed in the mid-1930s is generally considered the exemplar of this dominant design. The DC3 is an all-metal, monocoque, piston-propeller monoplane with twin engines placed under the wings. Production costs of this design rapidly fell due to its commercial success in both military and civil aviation. In the early 1940s, total production of the DC3 reached 10 000 models (Jane's 1978). The DC3 design also provided the basis of the development of a whole product family developed throughout the 1940s and the 1950s, including the DC4, DC5, DC6 and DC7. At the time, many firms, including Boeing, imitated the DC designs in their piston propeller product lines for passenger aircraft and bombers.

The third stage, covering the period of the 1940s and 1950s, is characterized by the introduction of jet engines. The first experiments with jet engines go back to the Second World War, but their successful application in both military and civil aircraft first took place in the 1950s. The transition from piston-propeller to jet engines has been widely recognized as a technological revolution, which has established a shift in the prevailing 'technological paradigm' (Constant 1980, Dosi 1982). The introduction of jet engines did not simply replace piston-propeller engines in existing designs, but also led to the development of new technologies in other parts of the aircraft, notably the introduction of swept and delta wings that were better able to cope with the increased engine power of jet engines. The revolutionary nature of jet-engine technology can be further supported by the fact that the Douglas, as the most successful company in large piston-

propeller aircraft, lost its leading position to Boeing, a company that came to dominate the turbofan passenger aircraft industry.

The fourth stage of aircraft development has been characterized by the further diffusion of jet engines in smaller aircraft, including business aircraft and short-range passenger aircraft. In the period after the 1950s, no major change in aircraft design has taken place as innovative activities increasingly shifted from aircraft design to avionics.

4.2.2   Aircraft data

The data on aircraft design concern the alleles of six design dimensions and covers the period 1913–84. As aircraft development only took off in the early 1900s, the data can be considered to cover the larger part of aircraft history. The choice of the six dimensions and its alleles is based on the limitation posed by the data source, which concerns photographs of aircraft designs. Admittedly, other dimensions that are known to have played an important role, including the types of landing gear and the type of materials used, could not be coded due to the limitation of the source materials. The photographs were drawn from Jane's (1978, 1989) encyclopaedia on aviation, which is known to be among the most comprehensive encyclopaedias of aviation and aircraft designs from all countries. The data of the six dimensions have been compiled for a sample of 731 aircraft models (Table 5.3), corresponding to a sample covering other variables not used here, previously assembled by Paolo Saviotti.[29]

The frequency distributions of designs that are used to measure entropy and mutual information at particular moments in time are not the yearly distributions of product designs. In this case, a year is too short a time-span, as aircraft designs are typically products that remain on offer for many years after their introduction. We used ten-year distributions, but calculations for five-year and 15-year distributions yielded the same trends as discussed below.

The results in the figures below are shown still using a yearly basis, where each year covers a ten-year period. Thus the distribution of designs associated with a specific year corresponds to a time period of ten years beginning in that year. In other words, the year 1913 stands for the distribution of designs introduced between 1913 and 1922; the year 1914 stands for the distribution of product designs introduced between 1914 and 1923, and so on.

4.2.3   Results on aircraft

The results on entropy and mutual information for aircraft are given in Figure 5.7. Entropy increased in the early decades and decreased only slightly in the 1930s. In the 1940s and early 1950s entropy increased rapidly, again to level

Table 5.3   Design space of aeroplane technology (S = 2520)

Aeroplane	
Number of observations:	*731*
Time span:	*1913–84*
Area:	*World*
X_1 $A_1 = 5$	Engine type 0 piston-propeller, 1 turboprop, 2 jet, 3 turbofan, 4 rocket
X_2 $A_2 = 7$	Number of engines 0 one, 1 two, 2 three, 3 four, 4 six, 5 eight, 6 twelve
X_3 $A_3 = 3$	Number of wings 0 monoplane, 1 biplane, 2 triplane
X_4 $A_4 = 4$	Wing type 0 straight, 1 delta, 2 swept, 3 variable swept
X_5 $A_5 = 2$	Number of tails 0 one, 1 two
X_6 $A_6 = 3$	Number of booms 0 one, 1 two, 2 three

off in the late 1950s. Mutual information shows fewer fluctuations, with a general upward trend. Notably, mutual information rose substantially during the period of the 1940s and 1950s and levelled off thereafter. The results suggests that the long-term evolution of aircraft is characterized by both growing variety and growing differentiation into different design families.

The results for the pair-wise mutual information in Figure 5.8 prove informative with respect to the dimensions along which the differentiation process has taken place. It is clear that the rise in mutual information in the post-war period is primarily related to rising mutual information between the engine type and the wing type $T(X_1,X_4)$, between the engine type and the number of engines $T(X_1,X_2)$, and between the number of engines and the wing type $T(X_2,X_4)$. The values for these three pairs of design dimensions have increased very rapidly. The emergence of design families can thus be related to the interdependencies between these design dimensions. The local optima in fitness landscapes are thus primarily characterized by the different alleles engine type, wing type, and the number of engines. Counting the various designs in the final period after 1960 leads us to distinguish between four design families (Frenken 2001): one- and two-engine piston-propeller aircraft with straight wings, two-engine turboprop mono-
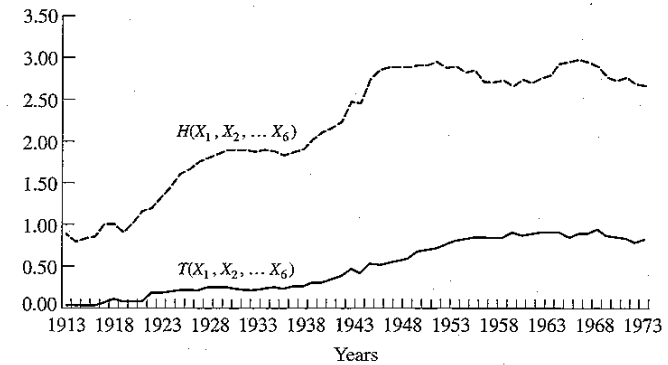
Figure 5.7   N-dimensional entropy (H) and mutual information (T) for aircraft designs
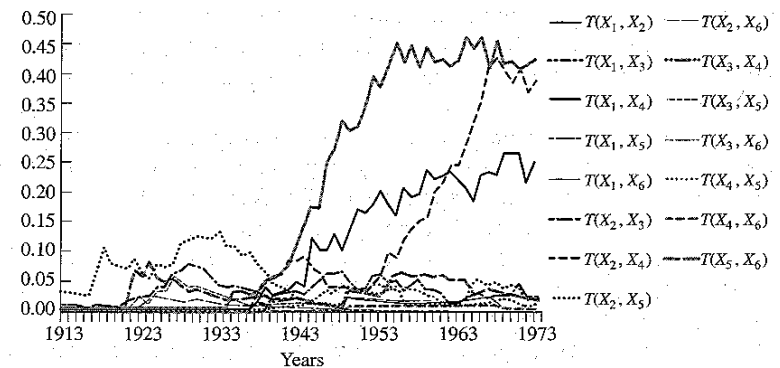
Figure 5.8   Two-dimensional mutual information (T) for aircraft designs

planes with straight wings, one- and two-engine jet aircraft with delta wings and two-, three- and four-engine turbofan aircraft with swept wings.

Epistatic relations among other pairs of dimensions do not show high dependence, suggesting that dimensions X_3, X_5 and X_6 have not been constitutive for the emergence of design families. All two-dimensional mutual information values including X_3, X_5 or X_6 remain low throughout the period with the exception of the value $T(X_2,X_5)$, This value shows some increase in the 1920s and early 1930s and indicates the common use of an uneven number of engines in two-tail aircraft design, with one engine placed

between the tails. After the 1930s, however, two-tail aircraft designs were hardly being used, which shows that this trajectory has proven a dead end.

4.2.4   Discussion

From our results we conclude that the history of aircraft technology is characterized by a progressive development of designs into four distinct families. Though not entirely differing with the histories of the aircraft industry as sketched before, these results offer a number of new insights into its evolutionary dynamics.

First, the emergence of a dominant design in the 1930s commonly associated with the Douglas DC3 had only a limited effect on the total design variety in the industry. The results on aircraft entropy show that the increase in variety was indeed halted during the 1930s, but did not decrease substantially. Second, the advent of jet-engine aircraft in the 1940s and 1950s contributed, as expected, to design variety, with entropy values rapidly rising during this period. However, after the 1950s entropy remained at a high level, suggesting that jet-engine design did not fully substitute propeller designs. Instead, a pattern of differentiation occurred, as indicated by the rising values of mutual information, with piston-propeller and turbo-propeller engine design coexisting alongside jet and turbofan engine designs.

We understand this stable pattern of differentiation as reflecting the different uses of different aircraft designs found in an earlier study that related engine types to market applications (Frenken 2000). Piston-propeller engine design has become dominant in low-cost, small-distance operations including trainer aircraft, business aircraft and agricultural aircraft. Turbo-propeller engine aircraft are used for small-distance passenger aircraft and military transport, while turbofan-engine aircraft are used for medium- and long-distance passenger aircraft. Finally, jet engines are predominantly used in high-speed fighter aircraft.

Note that the history of aircraft technology shows some interesting parallels with early steam-engine technology in that both technologies have witnessed the introduction of a revolutionary design (the jet engine and Watt's engine, respectively). Yet, in both industries the introduction of the revolutionary design has not so much led to a substitution process, but rather to a process of progressive differentiation into different design families.

4.3   Helicopters

4.3.1   Helicopter history

Though the concept of helicopters has a long history that goes back to China in about 400 BC, the first successful helicopter dates back to 1939

with the development of the VS-300 by Sikorsky (Taylor 1995). The advent of helicopter technology quickly received interest from armies and navies, because of helicopters' capacity to evacuate people from areas that were not accessible by aeroplanes. The military demand for helicopters induced a great deal of explorative activity in the 1940s and 1950s, including variations in the type of engine, the number of rotors, and the number of blades. At the time, commercial expectations were high, as evidenced by popular magazines predicting that American households would soon have a family of helicopters in the garage.

In the late 1950s, the explorative stage of technological development largely came to an end as design convergence took place with the apparent superior engine performance of turbines to piston engines. According to Bilstein (1996; p. 91), the single-rotor twin-turboshaft Kaman model introduced in 1954 can in hindsight be considered a 'pioneering' design. Hereafter, the twin-engine turboshaft design with one rotor became the 'dominant design'.

Commercially, however, helicopters never became a mass-produced product. Compared to aircraft, the costs and limited range of helicopters impede their wider diffusion in segments currently dominated by conventional aircraft (Taylor 1995). Instead, most helicopters are used for transporting people in areas not accessible by aircraft (such as military troops or offshore oil-platform personnel), while niche applications exist for a variety of uses, including ambulance operations and fighter operations.

4.3.2   Helicopter data

The data on helicopters concern the alleles of five design dimensions and cover the period 1940–83. The data of the five dimensions have been compiled for a sample of 144 helicopter models (Table 5.4). As for the data on aircraft, the helicopter data have been compiled on the basis of observable characteristics on photographs and correspond to the sample previously compiled by Paolo Saviotti[30] from Jane's (1978, 1989) encyclopaedia on aviation.

As for aircraft, the frequency distributions of designs that are used to measure entropy and mutual information at particular moments in time are not the yearly distributions of designs. We used again ten-year distributions, but the calculations for five-year and 15-year distributions yielded the same trends as in the results based on ten-year distributions discussed below. The results in the figures are shown per year, where each year stands for the first year of a ten-year period.

4.3.3   Results on helicopters

The results on entropy and mutual information are given in Figure 5.9. Interestingly, the results on helicopter variety and differentiation show

Table 5.4   Design space of helicopter technology (S = 420)

Helicopters	
Number of observations:	*144*
Time span:	*1940–83*
Area:	*World*
X_1 $A_1 = 5$	Engine type 0 piston, 1 piston turbo, 2 ramjet, 3 gas generator, 4 turboshaft
X_2 $A_2 = 3$	Number of engines 0 one, 1 two, 2 three
X_3 $A_3 = 7$	Number of blades 0 two, 1 three, 2 four, 3 five, 4 six, 5 seven, 6 eight
X_4 $A_4 = 2$	Number of shafts 0 one, 1 two
X_5 $A_5 = 2$	Number of rotors per shaft 0 one, 1 two

patterns that are altogether different from the results on early steam engines and aircraft. After a short period of rising values, entropy has fallen from 1955 onwards, showing that product variety in product designs has also fallen. The value for mutual information peaked earlier in 1949 and thereafter also shows a declining trend. Note that the decline in mutual information has been relatively greater than the decline in entropy values (mutual information halved during the period 1950–80). This suggests that the variety that remained was increasingly based on small variants around a single dominant design, which is the one-rotor turboshaft helicopter covering the large majority of models made from the 1950s onwards.

The two-dimensional mutual information values for helicopters also show decreasing trends (Figure 5.10). Only one pair of dimensions $T(X_2, X_3)$ shows the highest values over the whole period, reflecting complementarities between the number of engines and the number of blades. This relationship points to the common use of more blades when more engines are incorporated in a helicopter design to carry the higher weight. These variations remained within the dominant family of one-rotor turboshaft helicopters.

4.3.4   Discussion

The fall in mutual information accompanied by a fall in entropy suggests that after a brief period of differentiation, we have a prolonged phase in
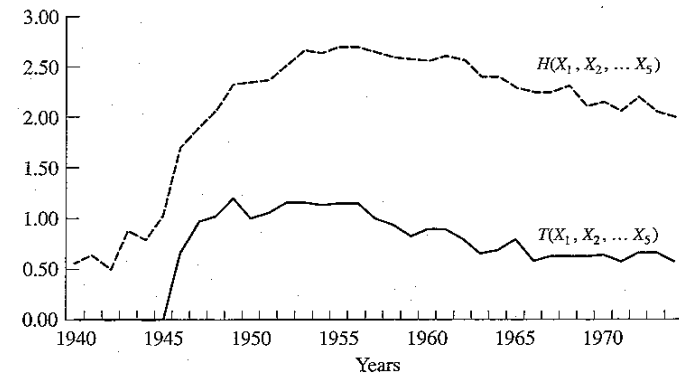
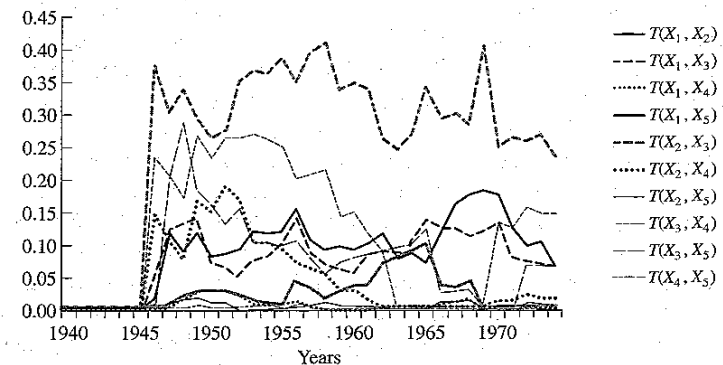*Figure 5.9   N-dimensional entropy (H) and mutual information (T) for helicopter designs*

Figure 5.10   Two-dimensional mutual information (T) for helicopter designs

which design variety decreases. The results correspond to Bilstein's (1996, p. 91) historical account that identified the single-rotor twin-turboshaft design as the dominant design emerging in the 1950s. Our analysis is also in line with findings by Saviotti and Trickett (1992, p. 116), who found that the single-rotor turboshaft helicopters increased their share in the population from around 30 per cent in the late 1950s to around 80 per cent in the early 1980s.

In the case of helicopter technology, de-differentiation cannot be attributed to absence of heterogeneity in demand. In fact, Saviotti and Trickett (1992) distinguish between up to 22 different uses of helicopters, ranging from fighter operations to military transport to ambulance to business transport. User heterogeneity may well be at least as high as in aircraft industry, even though sales in the helicopter industry are only a fraction of those in the aircraft industry. Given the heterogeneity of helicopter demand and the process of de-differentiation of helicopter supply, Saviotti and Trickett conclude that heterogeneity in demand is met by modular designs capable of being used in a variety of user contexts. In this context, one must think of helicopters in which the interior is easily adapted without changing the helicopter design itself.

The results still leave open the question as to why heterogeneity in user contexts in early steam-engine and aircraft design have triggered differentiation, while heterogeneity in helicopters users has not led to a sustained pattern of differentiation. Following Frenken et al. (1999b), one can explain the de-differentiation in helicopter technology by the existence of competition between helicopter technology and aircraft technology. Within the market for air transport, helicopter technology itself operates within a relatively small niche, which is bounded by the presence of aircraft technology. Over the past few decades, single-rotor helicopter performance has been limited by a flight range of around 1000 km, a speed of 300 km/h and payload of around 10000 kg. The halt in improvements does not reflect technical difficulties, but competition with aircraft: further improvements in speed, range or payload are technically perfectly realizable, but would lead helicopters to compete with small cheap aircraft covering the market segments of longer distances, higher speeds and higher payload. Put another way, the range of performance levels that helicopters are technically capable of reaching has not been fully explored due to the presence of cheaper aircraft technology.

5.   CONCLUSION

We started our study by introducing the NK model as a formal model of complex evolving systems that are characterized by interdependencies among their constituting components. We proposed a number of generalizations to the original NK model to account for the specificities of technological evolution. By examining the properties of this generalized NK model, we concluded that technological development in complex technologies is likely to lead to a process of differentiation of designs into distinct families. This view contradicts models of technological substitution that

depict competition among designs as a one-dimensional (cost-based) process that leaves room for only one surviving technology.

To analyse the evolutionary pattern of technological development in terms of changes in variety and differentiation, we proposed the methodology of entropy statistics. Entropy provides us with a comprehensive measure of design variety, while mutual information indicates to what extent this variety is non-random, that is, clustered in specific areas of the design space. The existence of multiple clusters indicates the presence of local optima in the technology's fitness landscape.

We applied the entropy statistics to data on design dimensions of three technologies. The results confirmed our hypothesis of increasing variety through differentiation for aircraft and steam engines, while the de-differentiation process of helicopter technology could be attributed to the presence of competing aircraft models. Furthermore, the empirical results offered us insights into the (quantitative) evolution that differ from the received histories of steam engines and aircraft. We found that the evolution of two technologies is better described as an evolutionary process of differentiation than as a linear substitution process. Obviously, a next step is to apply the methodology presented in this chapter to other technologies. The proposed methodology can be applied to any technology, given that sufficient empirical data are available on the relevant design dimensions of the technology in question.

APPENDIX: DERIVATION OF MUTUAL INFORMATION FOR ZERO ENTROPY AND MAXIMUM ENTROPY

Entropy is zero when one design occurs with frequency one, implying that the alleles incorporated in this design also occur with frequency one. Therefore, the sum of marginal entropy values equals zero, implying that mutual information equals zero:

$$T(X_1, \ldots, X_N) = \left(\sum_{i=1}^{N} H(X_i) \right) - H(X_1, \ldots, X_N)$$

$$T(X_1, \ldots, X_N) = \left(\sum_{i=1}^{N} -1 \cdot \ln 1 \right) - -1 \cdot \ln 1 = 0 + 0 = 0$$

Entropy is maximum when all possible designs in design space have an equal frequency $1/S$. In that case, the alleles along each dimension also have an equal frequency with marginal frequencies equalling $1/A_i$. Mutual information becomes:

$$T(X_1, \ldots, X_N) = \left(\sum_{i=1}^{N} H(X_i) \right) - H(X_1, \ldots, X_N)$$

$$T(X_1, \ldots, X_N) = \left(\sum_{i=1}^{N} \ln A_i \right) - \ln S$$

$$T(X_1, \ldots, X_N) = \ln \left(\prod_{i=1}^{N} A_i \right) - \ln S$$

$$T(X_1, \ldots, X_N) = \ln S - \ln S = 0.$$

NOTES

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1. Bradshaw (1992) uses the concepts of Simon to provide an interesting account of the Wrights' development of early aircraft technology.
2. See Kauffman and Macready (1995), Levinthal (1997), Frenken et al. (1999a), Auerswald et al. (2000), Gavetti and Levinthal (2000), Kauffman et al. (2000), Marengo et al. (2000), Rivkin (2000), Valente (2000), Fleming (2001), Fleming and Sorenson (2001), Frenken (2001). See also the discussion by Simon (2002) on the relationship between Kauffman's NK model and Simon's (1969 [1996]) early work.
3. Notable expectations are Fleming (2001) and Fleming and Sorenson (2001).
4. See also David (1975) in his discussion on localized technological change.
5. The combinatorial nature of the design space of a system requires that dimensions are orthogonal to one another. Therefore, one dimension of a system cannot correspond with an allele of another dimension in the same system. For example, the description of alleles of the engine dimension as gasoline ('0'), electric ('1') and steam ('2') implies that the type of battery used in electric engines cannot count as another dimension in the description of the vehicle as a system. The choice of a type of battery only constitutes a dimension for electric vehicles, and not for vehicle technologies in general.
6. Note that, since the first allele is labelled '0', the description of alleles of an element ranges from 0 to $A_i - 1$, while the number of alleles ranges from 1 to A_i.
7. A system's architecture has also been termed the system's internal structure (Simon 1969 [1996], Saviotti 1996).
8. The K value is an indicator of the complexity of a system's architecture and does not exactly coincide with the system's computational complexity, which can be expressed as the computational time that is required to globally optimize a complex system. On this, see Frenken et al. (1999a).
9. Allowing for mutation in several dimensions at the same time would permit a designer to escape local optima. However, the more dimensions that are allowed to be mutated at the same time, the higher the search costs involved as the number of possible moves increases exponentially with the number of dimensions that is allowed to be mutated at the same time. One can thus argue that designers are expected to search in only a few dimensions at the same time. On this issue, see Frenken et al. (1999a) and Kauffman et al. (2000).

10. For more properties of the NK model, see Kauffman (1993), Altenberg (1997), and Frenken et al. (1999a).
11. This perspective on fitness differs from the NK model applied to process technology where fitness is expressed only by a single cost criterion (Auerswald et al. 2000, Kauffman et al. 2000).
12. Altenberg's generalized NK model also allows one to model search by adding new components to a system, increasing N while keeping the number of selection criteria F constant. On this, see Altenberg (1994, 1995).
13. This is a relatively simple function sometimes applied in multi-criteria analysis of project selection (Nijkamp et al. 1990). This function implies that a loss in fitness of one function can infinitely be substituted by an increase in other functions. Various alternative functions exist to derive a fitness value or 'utility' from a collection of characteristics (Lancaster 1966, 1979).
14. Compare the SCOT approach of Pinch and Bijker (1984), who stress the interpretative flexibility of the meaning and use of artefacts. Here, the sociology of technology meets evolutionary economics.
15. Note that in the case of heterogeneous user groups, some weights can equal zero, while in the case of a homogeneous user group in formula 5.7, all weights are by definition positive. A zero weight in a homogeneous user population would imply that the feature does not count as a function for anyone.
16. In information theory entropy is computed using the logarithm of two instead of the natural logarithm taken here (Theil 1967, 1972, Frenken et al. 1999b).
17. $0 \cdot \ln(0) \equiv 0$.
18. A more elaborated account can be found in Frenken and Nuvolari (2002).
19. In this respect, Dickinson (1938) can be considered an exemplary reference.
20. A number of Newcomen engines were successfully used to raise water over a water wheel which, in turn, delivered rotary motion for factory machinery. These types of engine were usually called returning engines.
21. Von Tunzelmann (1978, p. 263).
22. For more details on the original data see Kanefsky (1979). For a more accessible reference, see Kanefsky and Robey (1980).
23. To be more precise, apart from the Newcomen design a second engine design was available before 1760. This design is the Savery engine, which we have excluded altogether from the analysis as it did not meet the classification of our design space. We consider the Savery engine to be a steam pump rather than a steam engine as it lacks the characteristic piston–cylinder arrangement characteristic of all the other steam engines. The exclusion of the Savery engine should not affect our results since only 33 Savery engines are present in the original data. More details on the Savery engine can be found in Frenken and Nuvolari (2002).
24. A similar conclusion based on historical grounds, stressing the role of variety, has been reached by Von Tunzelmann (1978, p. 24): 'It is misleading to see the pattern of progress [in steam-engine technology] as linear and inevitable: in explaining the direction and the chronology of "technical progress" in the economist's sense, it is vital to keep this diversity in mind.'
25. Joseph Bramah stated that the Newcomen engine had over Watt 'an infinite superiority in terms of simplicity and expense'. John Smeaton, one of the leading engineers of the time, considered that the Watt engine demanded too high standards for construction and maintenance. See Harvey and Downs-Rose (1980, pp. 22–3).
26. See Hills (1970, pp. 179–86). Many contemporary engineers believed that the rotary drive produced by a water-returning engine was much more regular and, in the end, 'better' than the one obtained from a rotary Watt engine. See also von Tunzelmann (1978, pp. 142–3).
27. On the Symington engine see Harvey and Downs-Rose (1980, ch. 3).
28. Dickinson and Jenkins (1927, p. 318). See also Farey (1827, p. 656).
29. See Saviotti and Bowman (1984) and Saviotti (1996).
30. See Saviotti and Trickett (1992) and Saviotti (1996).

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6. Complementarity constraints and induced innovation: some evidence from the first IT regime

Andreas Reinstaller and Werner Hölzl

1. INTRODUCTION

Schumpeter (1939) distinguished three stages in the process of technical change: (i) invention, that is, the act of creation of a new technology, (ii) innovation, its commercial introduction, and (iii) diffusion, its gradual adoption. Evolutionary economists recognized the importance of Schumpeter's trichotomy, but in the past their work has mostly focused on the last two stages of the process. The inducements and focusing devices leading entrepreneurs to produce new combinations are not analysed in an appropriate way. Technological search is often depicted as random. Neoclassical work on technical change has long studied John Hicks's induced innovation hypothesis in the framework of aggregate production functions. The key insight is that 'a change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind directed to economizing the use of a factor which has become relatively expensive' (Hicks 1932, pp. 124–5). This literature studies the inducement mechanism relying on the principle that a rise in real wages will trigger labour-saving innovation. The problem arising from this type of work is that the aggregate production function framework seems not to be appropriate, as technological change is an inherently microeconomic phenomenon. Second, neoclassical production functions of the Cobb–Douglas, CES or translog type are strongly separable. Separability amounts to the claim that the marginal rate of substitution of any pair of inputs is unaffected by changes in the level of another input.[1] Inputs or groups of inputs cannot be complementary. As the innovation process is not only a microeconomic phenomenon but also determined by the systemic character of firms and the technology they use, this assumption is quite strong.

The aim of this chapter is to analyse how recombinant search is triggered, how it is done and how initial conditions influence the final design