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On shadow prices for the measurement of sustainability

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Abstract The objective of this research is to discuss the direction of the bias of the existing estimates of genuine savings (also known as adjusted net savings). Such estimates rely on observed prices and quantities of investment and natural resource extraction. This has two consequences: first, it causes an overestimation of the shadow price of productive natural resources; second, it leads to omitting the depreciation of environmental services and amenities. We use simple numerical models to determine the path of optimal development under different assumptions. We find that the existing estimates of genuine savings are likely to be biased upward for countries with high levels of pollution, and biased downward for natural resource extracting countries.

Keywords Sustainable development · Genuine savings · Shadow prices

1 Introduction

During the last two decades, a growing body of literature has focused on defining and measuring the sustainability of economic development.[1]

Economists have broadly relied on simple dynamic models based on intertemporal social welfare functions with a constant discount rate. Research has produced an indicator of sustainability currently accepted by most scholars, named genuine savings, given by the

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[1] For recent surveys of the literature, see Asheim (2003, 2007a, b), Dasgupta (2001), Dasgupta and Mäler (2004), Hamilton and Whithagen (2007) and Pezzey and Toman (2002).

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sum of the quantities of net investments in all forms of capital, valued at appropriate shadow prices.

If the definition of capital is exhaustive—i.e. if it includes all existing physical, human, social and natural assets—and if shadow prices are constant within each period—i.e. if they exclude capital gains—genuine savings are informative on sustainability in two ways. First, they provide a two-way test of local sustainability, defined as non-decreasing intertemporal social welfare. Positive genuine savings indicate that the future generation will enjoy a larger stock of productive assets—implying higher intertemporal social welfare—relative to the current one. Symmetrically, negative genuine savings indicate that the set of productive assets is decreasing, and a smaller set of opportunities is left as bequest to those who follow.

If a global definition of sustainability is adopted, i.e. non-decreasing levels of utility at any future time on the path of optimal development, genuine savings allow performing only a one-way test of sustainability (Pezzey 2004). More specifically, negative genuine savings imply that utility will sooner or later drop below the current level. On the contrary, positive genuine savings do not guarantee that, on the optimal development path, utility will always be greater than today.

The objective of this research is to discuss the validity of the estimates of genuine savings presented in the existing empirical literature. More specifically, we aim to determine the direction of the bias due to overlooking the full set of interactions between natural resources and consumer utility.

The shadow prices of the stocks of physical, human, social and natural assets are key elements for the measurement of genuine savings. In general, the shadow price—measured in units of utility—is equal to the present value of the changes in the flow of future utilities due to an increment of one unit in the current stock. Mathematically, in the theory of optimal control, shadow prices are given by the values of the costate variables when the Hamiltonian is maximized.

When the use of a natural resource is associated with the generation of pollution, its shadow price decreases. The reduction in its stock represents a smaller loss for the future generation, as it will be accompanied by lower polluting emissions. Several articles discuss the problem from a theoretical point of view (Hartwick 1990; Hamilton and Clemens 1999; Hamilton 2000, 2003; Heal 1998; Neumayer 2003; Perman et al. 2003). However, the empirical literature surprisingly overlooks the point. The existing estimates of genuine savings (e.g. the Adjusted Net Savings of the World Bank) assess the depreciation of the stock of productive natural capital using the observed rent, given by the difference between observed market price and extraction cost[2] (World Bank 1997, 2005). Omitting to subtract the value of the environmental damage from the market price leads to underestimating genuine savings—a counterintuitive result, as noted by Neumayer (2003). The underestimation is particularly relevant for natural resource extracting countries, for which the World Bank's Adjusted Net Savings are at times negative—a result that originated a broad literature on the natural resource curse (see for example Atkinson and Hamilton 2003).

To explore the consequences of overlooking the interaction between environment and utility on the estimates of genuine savings, we compare three simple theoretical models. In the first, the interaction between environmental quality and utility is omitted. Welfare maximization determines the path of development chosen by an economy in which the environment is external to the markets. As the empirical literature relies on observed market prices and quantities, and in real-world economies environmental effects are rarely

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[2] Average rather than marginal extraction costs are considered, as information on the latter is unavailable.

internalized, we argue that existing measures of genuine savings are consistent with the solution of this model.[3] In the second model, extraction and employment of natural resources for production negatively affect an otherwise constant flow of environmental services, on which consumer utility depends. The two models share production technology and size of capital stocks, and differ only in the expression for the utility of the representative consumer—in one case function of produced goods only, in the other of produced goods and environmental services. Because of the different utility function, all quantities (produced, extracted, consumed, etc.) and shadow prices differ on the paths of optimal development consistent with the two models. The comparison of the two solutions allows isolating the error in the choice of the path of optimal development due to ignoring the connection between production and consumption, via pollution. Eventually, in the third theoretical model we relax the hypothesis of a constant flow of environmental services by introducing a second stock of environmental goods, which provide amenities directly to consumers. The pursuit of optimal development implies making decisions on three sets of assets: man-made capital and productive natural resources—like in the first two models—and environmental amenities.

We derive the analytical expressions for shadow prices and genuine savings for the three models. In the first two, genuine savings miss one term related to the decrease in the stock of environmental amenities. Further comparisons are not straightforward because each model determines different paths of optimal development, with different values for all variables (quantities, productivities, marginal costs, marginal utilities, etc.). For this reason, also when the expressions are formally identical, their values will generally differ. For example, on the optimal path, in all the models the shadow price of produced capital is equal to the marginal utility of consumption; however, unless we assume constant marginal utility of consumption, the values will differ in each model. We, nonetheless, attempt to determine under which conditions omitting the environment leads to the overestimation of the shadow price of productive natural resources.

To overcome the difficulties in the comparison of the analytical expressions for genuine savings, we develop a computable version of the three models using GAMS. We employ specific functional forms for production and consumer utility, and determine the path of optimal development and the optimal value of control variables (consumption and extraction of productive natural resources and environmental amenities), stocks (man-made capital and natural resources), and shadow prices at each period t.

The comparison between the optimal solutions of the three models shows that ignoring negative external effects on utility leads to a faster exploitation of productive natural resources. Symmetrically, the shadow price of productive natural resources is overestimated, as the marginal environmental damage is not deducted. Overestimating the optimal quantity of extraction and the shadow price of the relative stock leads to underestimating genuine savings. This supports the intuition—previously introduced—that existing estimates of genuine savings for resource-extracting countries may be excessively pessimistic.

Overlooking the relationship between consumer utility and environmental amenities leads to an overestimation of genuine savings—because of the omission of amenities depreciation. Most existing estimates of genuine savings attempt to account for the social costs of pollution. For example, the World Bank's Adjusted Net Savings subtract the value

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[3] Although the expression for genuine savings is formally identical to the one obtained from a model in which consumers' utility is also a function of the stocks of environmental resources, in this latter case quantities and prices for the calculation of genuine savings would be different from those observed in an economy in which the environment is external to the markets.

of the environmental damage due to the emissions of particulates and carbon dioxide. However, such correction is affected by two kinds of bias. First, observed quantities of polluting emissions exceed those that would be observed on the path of optimal development, as the lack of Pigouvian taxes leads to higher levels of emissions. Second, the shadow price differs from the one on the optimal development path. If the estimates are based on the marginal willingness to pay or accept, the price is overestimated, as emissions exceed the optimal quantity and marginal environmental damages have positive first and second derivatives. On the contrary, the price is underestimated if abatement costs are considered, as observed abatement is smaller than the optimal value and only the cheapest units of pollution are being prevented.

The rest of this article is organized as follows. Section 2 presents the theoretical models, derives the conditions for a path of optimal development, compares the expressions for genuine savings, and attempts to analytically determine the direction of the bias due to ignoring the relationship between consumer utility, environmental services and production. Section 3 introduces the numerical models, and discusses the difference between the paths of optimal development and the measures of genuine savings, in light of the analytical results from Sect. 2. Section 4 concludes.

2 The path of optimal development in three alternative theoretical models

Assume the existence of two types of environmental goods. The first is a productive input. Its use generates pollution, and therefore reduces environmental quality. The second produces amenities that increase consumer utility. No externality is internalized, so that the environment is considered only as provider of inputs for production. Utility in each period is a function of commodity consumption (C_t) only. Production is a function of the stock of physical capital (K_t) and of the amount of productive natural resource extracted in the period (R_t). For simplicity, assume that physical capital does not depreciate, that R is not renewable and that its extraction is not costly. The functions of utility and production are time invariant. The social welfare maximization problem can be written as follows (model 1):

$$\begin{aligned} \max_{C,R} W &= \int_t^{\infty} U(C_s)\mathrm{e}^{-r(s-t)}\mathrm{d}s \text{ s.t.} \\ \dot{K}_t &= F(K_t, R_t) - C_t \\ \dot{S}_t &= -R_t \end{aligned} \tag{1.1}$$

The initial stocks of K and S are exogenously determined ($K_0 = K0$, $S_0 = S0$). The following holds: $U_C > 0$, $U_{CC} < 0$, $F_K > 0$, $F_{KK} < 0$, $F_R > 0$, $F_{RR} < 0$. The current value Hamiltonian (at time t) and four necessary conditions for an optimum are[4]:

$$H = U(C) + \gamma^k \cdot [F(K,R) - C] + \gamma^S \cdot [-R] \tag{1.2}$$

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[4] The variables H, C, K, R, S, all the shadow prices γ and the derivatives of U and F with respect to C, K and R change every period and should be indexed by time. To simplify the notation, in what follows we omit the subscript t.

$$\begin{aligned} \gamma^K &= U_C \\ \gamma^S &= U_C F_R \end{aligned} \tag{1.3}$$

$$\begin{aligned} \frac{\dot{\gamma}^K}{\gamma^K} &= r - F_K \\ \frac{\dot{\gamma}^S}{\gamma^S} &= r \end{aligned} \tag{1.4}$$

Equations 1.3 define the expressions for the shadow prices of man-made capital and productive natural resources. The first states that, on the optimal path, in each period the shadow price of the stock of man-made capital K equals the marginal utility of consumption. According to the second, the shadow price of the stock of the productive natural resource equals the utility given by the consumption of the marginal product of one unit of S.

Equations 1.4 define the dynamic of the shadow prices. The first, combined with (1.3), states that on the optimal path of development the marginal utility of consumption grows at a rate which compensates for the rate of intertemporal preference, net of the return of the investment of one unit of capital—i.e. net of the marginal product of capital. The second states that the shadow price of the stock of productive natural resources grows at a rate that compensates intertemporal preference. This is the well-known Hotelling rule (Hotelling 1931) in the special case of costless extraction and can be written as follows:

$$\frac{\dot{\left(U_C F_R\right)}}{U_C F_R} = r \tag{1.5}$$

Some algebraic manipulation provides the expression for the optimal growth of the marginal product of R:

$$\frac{\dot{F}_R}{F_R} = F_K \tag{1.6}$$

The growth rate of the marginal product of the natural resource (F_R) equals the marginal product of capital (F_K), hence it is always positive. At least if the production function is separable in K and R, this implies that natural resource extraction is highest in the first period, and then decreases across time.

Genuine savings measured in units of consumption[5] are equal to the sum of the value of the change in all forms of assets, in this case man-made capital and productive natural capital, and are given by the following expression:

$$\frac{\mathrm{GS1}}{U_C} = [F(K,R) - C] + F_R \cdot [-R] \tag{1.7}$$

The first addendum of the right hand side of expression (1.7) is the change in the quantity of man-made capital, equal to production minus consumption. Its shadow price is normalized to one. The term in square brackets in the second addendum is the change in the stock of natural resources, equal to the extraction in the period. Its shadow price—normalized by the marginal utility of consumption—is equal to the marginal product of the natural resource (which measures the growth in utility due to the extraction and use of one more unit of natural resource).

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[5] The normalization is equivalent to using the shadow price of physical capital as numeraire.

In order to understand the bias due to ignoring environmental externalities, we consider a different model. Consumer utility is now a function of commodity consumption and of a flow of environmental services (E_t). Examples of environmental services are carbon sequestration, free and clean fresh water, sites for outdoor activities, clean air (affecting the number of sun-light days per year), etc. The flow of environmental services is negatively affected by the amount (R_t) of the stock S_t extracted for production purposes. The rest of the model is identical to model 1. The social welfare maximization problem can be written as follows (model 2):

$$\begin{aligned} \max_{C,R} W &= \int_t^{\infty} U(C_s, E(R_s))\mathrm{e}^{-r(s-t)}\mathrm{d}s \text{ s.t.} \\ \dot{K}_t &= F(K_t, R_t) - C_t \\ \dot{S}_t &= -R_t \end{aligned} \tag{2.1}$$

The initial stocks of K and S are exogenously determined ($K_0 = K0$, $S_0 = S0$). In addition to the conditions on first and second derivatives stated for model 1, the following holds: $U_E > 0$, $U_{EE} < 0$, $E_R < 0$, $E_{RR} < 0$. The current value Hamiltonian and four necessary conditions for an optimum are:

$$H = U(C, E(R)) + \gamma^k \cdot [F(K, R) - C] + \gamma^S \cdot [-R] \tag{2.2}$$

$$\begin{aligned} \gamma^K &= U_C \\ \gamma^S &= U_C F_R + U_E E_R \end{aligned} \tag{2.3}$$

$$\begin{aligned} \frac{\dot{\gamma}^K}{\gamma^K} &= r - F_K \\ \frac{\dot{\gamma}^S}{\gamma^S} &= r \end{aligned} \tag{2.4}$$

Three of the four first-order conditions for an optimum are formally identical to model 1. The specification "formally" is important, because the proper consideration of environmental externalities implies that the value of consumption and extraction chosen every year on the path of optimal development will be different from model 1. Furthermore, the shadow price of the stock of productive natural resources is now reduced by the utility cost of the environmental damage caused by production. The fact that the productive use of natural resources determines a negative externality on environmental services makes natural resources, *ceteris paribus*, less valuable.

Substituting the static conditions into the dynamic ones and rearranging we obtain interesting insights on the difference between the two paths of optimal development. In both models, the rate of growth of the marginal utility of consumption equals the difference between rate of intertemporal preference and marginal product of capital. In both models, the rate of growth of the shadow price of productive natural resources equals the rate of intertemporal preference. For model 2, we have the following expression:

$$\frac{\left(U_C F_R \dot{+} U_E E_R\right)}{U_C F_R + U_E E_R} = r \tag{2.5}$$

Some algebraic manipulation provides the expression for the optimal growth of the marginal product of the natural resource.

$$\frac{\dot{F}_R}{F_R} = F_K + \frac{U_E E_R}{U_C F_R} \cdot \left(r - \frac{(\dot{U_E E_R})}{U_E E_R} \right) \tag{2.6}$$

In model 1, the rate of growth of F_R equals F_K, hence it is always positive. If the production function is separable in K and R, this implies that the extraction R is highest in the first period and decreases across time.[6] The same does not necessarily hold in model 2, where the marginal product of R can decrease across time if the second term on the right hand side of expression (2.6) is negative and sufficiently large. As $\frac{U_E E_R}{U_C F_R}<0$, a necessary condition for this to happen is $\frac{(\dot{U_E E_R})}{U_E E_R} < r$—i.e. the growth rate of the marginal environmental damage be smaller than the rate of intertemporal preference. This can happen if the former is negative, or is positive but sufficiently small. In model 2, it is possible to envisage a scenario in which the extraction of productive natural resources is initially small, and then grows across time. On the contrary, a positive and high growth rate of marginal externalities is associated with an increase in the marginal product of R, and reinforces the result of model 1, with large initial extraction that rapidly decreases across time.

Genuine savings—in units of consumption—are given by the following expression:

$$\frac{\mathrm{GS2}}{U_C} = [F(K,R) - C] + \frac{(U_C F_R + U_E E_R)}{U_C} \cdot [-R] \tag{2.7}$$

Formally, the expression broadly replicates the one obtained from model 1. The only difference lies in the shadow price of productive natural resources.

Assume that the solution of the maximization problem associated with model 2 provides insights on the optimal path of development. Imagine, on the other hand, that model 1 better describes the way in which the world economy actually works—because most relationships between production, environmental quality and utility are ignored by the markets. The theory tells us that development is not sustainable if the genuine saving measured on the path of optimal development is negative (Pezzey 2004). However, existing estimates of genuine savings are consistent with expression (1.7) from model 1 (GS1), rather than with expression (2.7). The estimates of GS1 differ from the correct measure GS2 because: (a) the quantities of production, consumption, extraction (and therefore the marginal utility of consumption, the marginal product of capital and natural resources) are not the ones one would have if the social planner was solving the correct maximization problem; (b) expression (1.7) fails to consider the fact that the shadow price of productive natural resources is reduced by the negative externalities on the utility of the representative consumer. In model 1, the use of productive natural resources is likely to be faster and the relative shadow price is likely to be higher than in model 2. Therefore GS1 is likely to represent an underestimation of the correct measure of genuine savings GS2.

Eventually, we consider a model in which the utility of the representative individual is also a function of a second stock of environmental resources that provide amenities directly for consumption. Utility in each period is a function of commodity consumption and environmental amenities—which are positively affected by the amount of environmental services M_t extracted from the stock Z_t of renewable environmental goods, and negatively affected by the amount R_t extracted for production purposes from the stock S_t. All other

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[6] If the production function is not separable in K and R, this does not necessarily hold, as the marginal product of R can grow even if the amount of extraction grows, if the stock of physical capital K is higher than in the first period.

assumptions are common to models 1 and 2. The social welfare maximization problem can be written as follows (model 3):

$$\max_{C,R,M} W = \int_{t}^{\infty} U(C_s, E(M_s, R_s))\mathrm{e}^{-r(s-t)}\mathrm{d}s \text{ s.t.}$$

$$\begin{aligned} \dot{K}_t &= F(K_t, R_t) - C_t \\ \dot{S}_t &= -R_t \\ \dot{Z}_t &= h(Z_t) - M_t \end{aligned} \tag{3.1}$$

Where $h(Z_t)$ is the function of natural renewal of the stock of environmental amenities. The initial stocks of K, S and Z are exogenously determined ($K_0 = K0$, $S_0 = S0$, $Z_0 = Z0$). In addition to the conditions on first and second derivatives stated for model 2, the following holds: $E_M > 0$, $E_{MM} < 0$. The current value Hamiltonian and six necessary conditions for an optimum are:

$$H = U(C, E(M, R)) + \gamma^k \cdot [F(K, R) - C] + \gamma^S \cdot [-R] + \gamma^Z \cdot [h(Z) - M] \tag{3.2}$$

$$\begin{aligned} \gamma^K &= U_C \\ \gamma^S &= U_C F_R + U_E E_R \\ \gamma^Z &= U_E E_M \end{aligned} \tag{3.3}$$

$$\begin{aligned} \frac{\dot{\gamma}^K}{\gamma^K} &= r - F_K \\ \frac{\dot{\gamma}^S}{\gamma^S} &= r \\ \frac{\dot{\gamma}^Z}{\gamma^Z} &= r - h_Z \end{aligned} \tag{3.4}$$

Four of the six first-order conditions for an optimum are formally identical to model 2. We find here two additional expressions. They state that the shadow price of the stock Z of environmental amenities is equal to the marginal utility produced by one unit of environmental amenity, and that this shadow price grows at a rate that compensates for the rate of pure intertemporal preference, net of the gain given by the natural growth of the stock of resource, which is a function of the existing stock.

Genuine savings in units of consumption are given by the following expression:

$$\frac{\mathrm{GS3}}{U_C} = [F(K, R) - C] + \frac{(U_C F_R + U_E E_R)}{U_C} \cdot [-R] + \frac{U_E E_M}{U_C} \cdot [h(Z) - M] \tag{3.5}$$

The first two terms of the expression for genuine savings are formally identical to the expression derived from model 2. However, there is now a third addendum, which measures the value of the change in the stock of environmental amenities. The quantity, in square brackets, is equal to the natural growth of the stock minus the extraction in the period. The shadow price is equal to the marginal utility given by exploiting one more unit of the existing stock.

As the comparison of quantities and prices for the measurement of genuine savings in the three models requires several assumptions of separability in the utility and production functions, and quickly becomes untreatable, we proceed in the next section with the analysis of the solution of equivalent numerical models.

3 Numerical models and the validity of existing measures of genuine savings

We develop and solve three discrete-time computational models—mirroring those discussed in Sect. 2—using GAMS. In these models, we assume specific functional forms for utility and production functions, and set the key parameters to somehow arbitrary values. Hence, the results we obtain do not hold in general. Nonetheless, running computational models can provide useful insights on the results of the optimization problem and on the value of genuine savings. In a second phase, we perform sensitivity analysis to test the robustness of our results.

The full specification of the three models and the detailed results of their solution are presented in Appendix 1 (Tables 3–5).

We focus first on the comparison of models 1 and 2, with the aim to single out the effect of the consideration of negative externalities from extraction and use of productive natural resources.

The comparison of the two sets of results (Tables 3 and 4 in Appendix 1) shows that, when externalities are not properly taken into account in model 1, productive natural resources are exploited more rapidly. This is shown in Fig. 1. Production and consumption are initially higher (Fig. 2). Also investments are higher, as a larger stock of man-made capital is required to compensate for exploited natural resources (and keep production high in the following periods). In both models, consumption increases with time, as the marginal product of capital exceeds the rate of intertemporal preference. In both models, net investments (production net of consumption and capital depreciation) become at some point negative. This is due to the fact that we consider a finite time horizon, and physical capital is progressively used for consumption purposes.

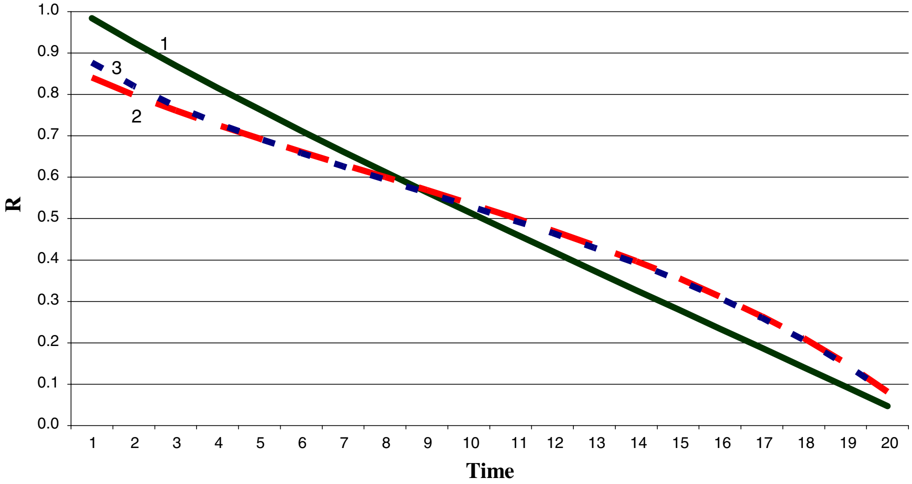

Fig. 1 Extraction R in the three models. *Note*: *Solid line* for model 1, *dashed* for model 2, *dotted* for model 3

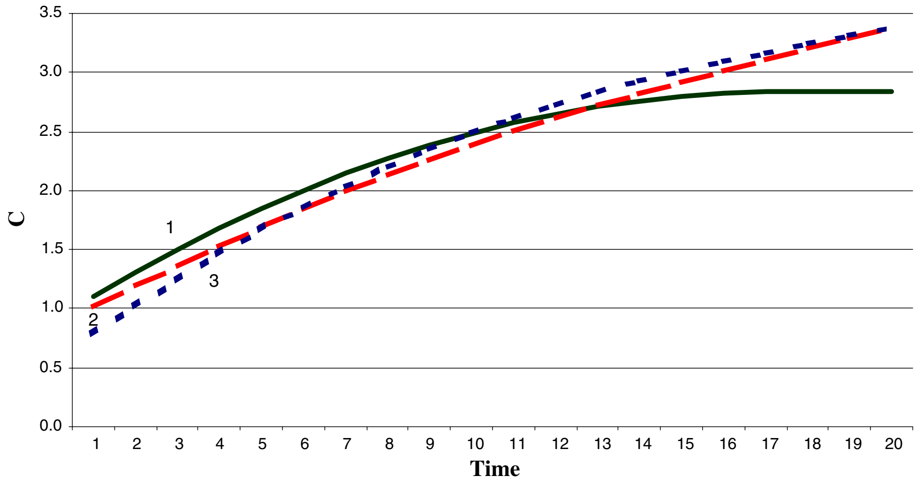

Fig. 2 Consumption C in the three models. *Note*: *Solid line* for model 1, *dashed* for model 2, *dotted* for model 3

The normalized shadow price of productive natural resources differs remarkably in the two models: more specifically, the negative externality on amenities makes the shadow price smaller in model 2.

The difference in the dynamic of productive natural resource extraction can be examined with reference to Eqs. 1.6 and 2.6—whose components are reported in Table 6 in Appendix 1. In model 1, optimal exploitation requires that the rate of growth of the marginal product of R equals the marginal productivity of man-made capital (as from Eq. 1.6). In model 2, optimal exploitation is also affected by the relationship between the rate of intertemporal preference and the growth rate of marginal environmental damages. The two are practically equal in the first period. Starting from the second period, the rate of growth of environmental damages is consistently smaller than the rate of intertemporal preference (and soon becomes negative), therefore the second addendum of expression (2.6) is negative. The marginal product of capital is consistently smaller in model 2 (in the first period, and for a number of periods afterward). The consideration of the interactions between pollution and environmental services makes the rate of growth of the marginal product of R smaller. This implies that natural resource extraction decreases more slowly across time—therefore, extraction is less intense in the first periods (see Fig. 1).

Summarizing, omitting environmental externalities leads to: (a) overestimating the optimal amount of extraction of productive natural resources; (b) overestimating their normalized shadow price; (c) overestimating the optimal quantity of investment in physical capital. Overall, in our simulations, ignoring environmental externalities leads to an estimate of genuine savings biased downward (Table 1). The sensitivity analysis presented in Table 2 shows that this result is robust to changes in the key parameters of the two models.

We next compare models 2 and 3. The main difference is that in model 2 the formula for genuine savings omits a negative term measuring the depreciation of the stock of amenity-providing environmental goods. This term is negative if harvest exceeds natural growth. *Ceteris paribus*, the omission biases genuine savings upward. Extraction and shadow price of productive natural resources do not seem to be substantially affected (see Fig. 1 for the quantity of extraction). As a consequence, production also does not change remarkably.

Table 1 Decomposition of genuine savings in the first period, in the three models

		ΔK	ΔS	ΔZ	GS
Model 1	Price	1.000[a]	1.594		−0.041
	Quantity	1.527	−0.984		
	Value	1.527	−1.568		
Model 2	Price	1.000[a]	1.206		0.371
	Quantity	1.385	−0.841		
	Value	1.385	−1.014		
Model 3	Price	1.000[a]	1.264	−0.29	0.182
	Quantity	1.681	−0.877	1.348	
	Value	1.681	−1.109	−0.391	

Note: [a] Shadow price of physical capital normalized to one

Table 2 Sensitivity analysis

Value of parameter	$\mathrm{GS1}/\gamma^K$	$\mathrm{GS2}/\gamma^K$	$\mathrm{GS3}/\gamma^K$
Baseline: $\alpha = \beta = 0.5$, $\delta = 0.05$, $r = 0.02$	−0.041	0.371	0.181
$r = 0.03$	−0.133	0.301	−0.004
$r = 0.01$	0.046	0.429	0.312
$\delta = 0.06$	−0.231	0.170	−0.112
$\delta = 0.04$	0.149	0.574	0.460
$\alpha = 0.04$	−0.129	0.418	−0.152
$\alpha = 0.06$	0.071	0.366	−0.378
$\beta = 0.04$	−0.925	−0.564	−1.254
$\beta = 0.06$ (requires $S_0 = 3$)	1.058	1.231	1.197

However, consumption is initially lower (see Fig. 2), determining a higher level of investment in man-made capital. This may be due to the need of a larger stock of man-made capital to compensate for the depletion of two stocks of natural resources. Overall, ignoring environmental amenities leads to: (a) underestimating the optimal quantity of investment in physical capital; (b) omitting the depreciation of the stock of amenity-providing environmental goods. Although the two errors bias genuine savings in opposite directions, the sensitivity analysis presented in Table 2 shows that genuine savings are consistently smaller in model 3 than in model 2.

4 Conclusions

The objective of this research was to discuss the validity of the estimates of genuine savings—the main indicator of sustainable development—presented in the empirical literature.

Existing estimates of genuine savings are based on observed market prices and quantities. As real-world markets do not internalize environmental externalities, such estimates are twice biased. First, the term measuring the depreciation of the stock of productive natural resources (for example oil) is overestimated. This is due to the fact that:

(a) extraction and use are faster in the real world than they would be on the optimal development path, because the price of pollution is not internalized; (b) the market price exceeds the shadow price on the optimal development path. The depreciation of the stock of productive natural capital actually represents a smaller loss for the future generation, because it will be accompanied by a reduced amount of polluting emissions. Relying on market prices leads to underestimating genuine savings. Second, a term measuring the depreciation of the stock of environmental goods providing amenities to consumers is omitted—as the market price is null. This omission leads to overestimating genuine savings.

The two biases have opposite signs. It is therefore not possible to determine the overall sign of the error. Nonetheless, our analysis shows that existing estimates of genuine savings are likely to be biased upward for countries with high levels of environmental damage from pollution, and biased downward for natural resource extracting countries.

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Appendix 1

This appendix presents three numerical models consistent with the theoretical ones discussed in Sect. 2. Model 1, which ignores the effect of environmental resources on utility, is specified as follows:

$$\begin{aligned} \max_{C_t,R_t} W &= \sum_{t=1}^{T} U_t \cdot (1+r)^{-t} \text{ s.t.} \\ K_{t+1} &= (1-\delta)\cdot K_t + Q_t - C_t \\ S_{t+1} &= S_t - R_t \\ C_T &= (1-\delta)\cdot K_T + Q_T \\ R_T &= S_T \end{aligned} \tag{4.1}$$

where

$$\begin{aligned} U_t &= C_t^{\alpha} \\ Q_t &= K_t^{\beta}\cdot R_t^{1-\beta} \end{aligned} \tag{4.2}$$

The model allows for depreciation of man-made capital, at a rate δ.

Model 2, which considers the negative external effects due to the extraction and use of productive natural resources, is specified as follows:

$$\begin{aligned} \max_{C_t,R_t} W &= \sum_{t=1}^{T} U_t \cdot (1+r)^{-t} \text{ s.t.} \\ K_{t+1} &= (1-\delta)\cdot K_t + Q_t - C_t \\ S_{t+1} &= S_t - R_t \\ C_T &= (1-\delta)\cdot K_T + Q_T \\ R_T &= S_T \end{aligned} \tag{5.1}$$

where

$$\begin{aligned} U_t &= C_t^{\alpha} \cdot E_t^{1-\alpha} \\ E_t &= \left(1 - \left(\frac{R_t}{2}\right)^{\mu}\right) \\ Q_t &= K_t^{\beta} \cdot R_t^{1-\beta} \end{aligned} \tag{5.2}$$

Model 3, which considers also the effect of environmental amenities on the utility of the representative consumer, is specified as follows:

$$\begin{aligned} \max_{C_t,R_t,M_t} W &= \sum_{t=1}^{T} U_t \cdot (1+r)^{-t} \text{ s.t.} \\ K_{t+1} &= (1-\delta) \cdot K_t + Q_t - C_t \\ S_{t+1} &= S_t - R_t \\ Z_{t+1} &= Z_t - M_t \\ C_T &= (1-\delta) \cdot K_T + Q_T \\ R_T &= S_T \\ M_T &= Z_T \end{aligned} \tag{6.1}$$

where:

$$\begin{aligned} U_t &= C_t^{\alpha} \cdot E_t^{1-\alpha} \\ E_t &= M_t^{\phi} \cdot \left(1 - \left(\frac{R_t}{2}\right)^{\mu}\right) \\ Q_t &= K_t^{\beta} \cdot R_t^{1-\beta} \end{aligned} \tag{6.2}$$

In all the models, we assume the following values for the parameters:

$T = 20$, $K_1 = S_1 = Z_1 = 10$, $\alpha = \beta = \phi = 0.5$, $\delta = 0.05$, $\mu = 2$, $r = 0.02$.

The first-order conditions for the solution of a complete discrete-time model are derived in Appendix 2. The results of the optimization are in Tables 3, 4 and 5. Table 6 reports the components of Eqs. 1.6 and 2.6, explaining the dynamic of the extraction of the natural resource.

Table 3 Optimal path of development—model 1

t	γ^K	γ^S/γ^K	C	Q	ΔK	R	GS/γ^K
1	0.475	1.594	1.109	3.136	1.527	0.984	−0.041
2	0.438	1.765	1.306	3.266	1.384	0.926	−0.249
3	0.409	1.926	1.496	3.351	1.210	0.870	−0.465
4	0.386	2.080	1.676	3.395	1.013	0.816	−0.685
5	0.368	2.226	1.845	3.400	0.798	0.764	−0.902
6	0.353	2.365	2.001	3.369	0.571	0.712	−1.113
7	0.341	2.496	2.144	3.305	0.336	0.662	−1.316
8	0.332	2.622	2.272	3.212	0.097	0.613	−1.509
9	0.324	2.741	2.387	3.090	−0.144	0.564	−1.689
10	0.317	2.854	2.487	2.943	−0.384	0.516	−1.856
11	0.312	2.961	2.574	2.771	−0.623	0.468	−2.009
12	0.307	3.063	2.647	2.577	−0.859	0.421	−2.148

Table 3 continued

t	γ^K	γ^S/γ^K	C	Q	ΔK	R	GS/γ^K
13	0.304	3.160	2.708	2.362	−1.092	0.374	−2.273
14	0.301	3.252	2.756	2.127	−1.321	0.327	−2.385
15	0.299	3.339	2.794	1.874	−1.546	0.281	−2.483
16	0.298	3.422	2.821	1.603	−1.766	0.234	−2.568
17	0.297	3.501	2.837	1.314	−1.983	0.188	−2.640
18	0.296	3.576	2.845	1.009	−2.197	0.141	−2.702
19	0.296	3.647	2.845	0.689	−2.407	0.094	−2.752
20	0.297	3.715	2.837	0.352	−2.615	0.047	−2.791

Notes: γ^K = marginal utility of consumption; γ^S/γ^K = shadow price of the stock of productive natural resources, in units of consumption; C = quantity consumed; Q = quantity produced; ΔK = quantity of net investment; R = quantity of extraction of productive natural resources; GS/γ^K = value of genuine savings, in units of consumption

Table 4 Optimal path of development—model 2

t	γ^K	γ^S/γ^K	C	Q	ΔK	R	GS/γ^K
1	0.450	1.206	1.015	2.900	1.385	0.841	0.371
2	0.420	1.321	1.194	3.015	1.252	0.799	0.197
3	0.395	1.430	1.368	3.100	1.101	0.761	0.013
4	0.376	1.534	1.536	3.158	0.935	0.726	−0.178
5	0.360	1.633	1.696	3.189	0.759	0.693	−0.373
6	0.347	1.729	1.850	3.194	0.573	0.661	−0.570
7	0.336	1.821	1.996	3.176	0.380	0.630	−0.767
8	0.327	1.911	2.134	3.134	0.181	0.600	−0.964
9	0.319	1.998	2.265	3.069	−0.024	0.569	−1.160
10	0.312	2.083	2.389	2.981	−0.236	0.537	−1.354
11	0.306	2.166	2.507	2.869	−0.454	0.505	−1.547
12	0.300	2.249	2.619	2.732	−0.680	0.471	−1.739
13	0.296	2.330	2.726	2.569	−0.915	0.435	−1.929
14	0.291	2.411	2.828	2.380	−1.161	0.397	−2.119
15	0.288	2.492	2.926	2.161	−1.420	0.357	−2.309
16	0.284	2.573	3.021	1.910	−1.695	0.312	−2.499
17	0.281	2.654	3.114	1.624	−1.989	0.264	−2.690
18	0.278	2.738	3.204	1.298	−2.306	0.211	−2.883
19	0.275	2.823	3.292	0.925	−2.651	0.151	−3.077
20	0.272	2.912	3.380	0.497	−3.034	0.082	−3.272

Note: See Table 3

Table 5 Optimal path of development—model 3

t	γ^K	γ^S/γ^K	C	Q	ΔK	R	γ^Z/γ^S	M	GS/γ^K
1	0.373	1.264	0.781	2.961	1.681	0.877	1.348	0.290	0.181
2	0.347	1.389	1.014	3.094	1.496	0.820	1.480	0.342	−0.148
3	0.327	1.503	1.243	3.189	1.287	0.772	1.602	0.388	−0.494
4	0.311	1.610	1.462	3.250	1.064	0.730	1.716	0.426	−0.842
5	0.299	1.710	1.670	3.280	0.834	0.693	1.822	0.458	−1.186
6	0.289	1.805	1.863	3.282	0.601	0.658	1.924	0.484	−1.519
7	0.280	1.896	2.041	3.258	0.369	0.626	2.021	0.505	−1.838
8	0.273	1.983	2.205	3.208	0.137	0.594	2.114	0.522	−2.143
9	0.267	2.068	2.354	3.135	−0.093	0.562	2.204	0.534	−2.433
10	0.262	2.150	2.491	3.037	−0.322	0.531	2.291	0.544	−2.709
11	0.258	2.230	2.615	2.915	−0.552	0.498	2.377	0.550	−2.971
12	0.254	2.309	2.729	2.769	−0.785	0.465	2.461	0.554	−3.222
13	0.251	2.387	2.833	2.598	−1.021	0.429	2.544	0.557	−3.463
14	0.248	2.465	2.929	2.400	−1.264	0.392	2.628	0.557	−3.694
15	0.245	2.543	3.017	2.173	−1.516	0.352	2.711	0.557	−3.919
16	0.242	2.622	3.099	1.915	−1.780	0.308	2.794	0.555	−4.137
17	0.240	2.702	3.176	1.623	−2.059	0.260	2.879	0.552	−4.350
18	0.237	2.783	3.248	1.293	−2.358	0.207	2.966	0.547	−4.558
19	0.235	2.867	3.314	0.919	−2.681	0.148	3.055	0.542	−4.761
20	0.233	2.953	3.375	0.492	−3.035	0.080	3.148	0.536	−4.958

Notes: See Table 3. γ^Z/γ^S = Shadow price of the stock of environmental amenities, in units of consumption; M = quantity of extraction of environmental amenities

Table 6 Dynamic of the extraction of productive natural resources in models 1 and 2, from Eqs. 1.6 and 2.6

t	Model 1			Model 2					
	$\frac{\dot{F}_R}{F_R}$	F_K	R	$\frac{\dot{F}_R}{F_R}$	F_K	$\frac{U_E E_R}{U_C F_R}$	$\frac{(\dot{U_E E_R})}{U_E E_R}$	$\frac{U_E E_R}{U_C F_R} \cdot \left(r - \frac{(\dot{U_E E_R})}{U_E E_R} \right)$	R
1	0.107	0.107	0.984	0.095	0.095	−0.301	0.019	0.000	0.841
2	0.092	0.092	0.926	0.080	0.082	−0.300	0.011	−0.003	0.799
3	0.080	0.080	0.870	0.067	0.073	−0.298	0.003	−0.005	0.761
4	0.070	0.070	0.816	0.058	0.065	−0.295	−0.003	−0.007	0.726
5	0.062	0.062	0.764	0.050	0.059	−0.290	−0.009	−0.009	0.693
6	0.056	0.056	0.712	0.043	0.054	−0.284	−0.015	−0.010	0.661
7	0.050	0.050	0.662	0.037	0.049	−0.277	−0.021	−0.011	0.630
8	0.045	0.045	0.613	0.032	0.046	−0.269	−0.028	−0.013	0.600
9	0.041	0.041	0.564	0.028	0.043	−0.260	−0.034	−0.014	0.569
10	0.038	0.038	0.516	0.024	0.040	−0.249	−0.042	−0.015	0.537
11	0.034	0.034	0.468	0.021	0.038	−0.238	−0.051	−0.017	0.505
12	0.032	0.032	0.421	0.018	0.036	−0.225	−0.061	−0.018	0.471

Table 6 continued

t	Model 1			Model 2					
	$\frac{\dot{F}_R}{F_R}$	F_K	R	$\frac{\dot{F}_R}{F_R}$	F_K	$\frac{U_E E_R}{U_C F_R}$	$\frac{\dot{(U_E E_R)}}{U_E E_R}$	$\frac{U_E E_R}{U_C F_R} \cdot \left(r - \frac{\dot{(U_E E_R)}}{U_E E_R}\right)$	R
13	0.029	0.029	0.374	0.015	0.035	−0.211	−0.074	−0.020	0.435
14	0.027	0.027	0.327	0.012	0.033	−0.195	−0.091	−0.022	0.397
15	0.025	0.025	0.281	0.009	0.032	−0.178	−0.113	−0.024	0.357
16	0.023	0.023	0.234	0.005	0.032	−0.158	−0.145	−0.026	0.312
17	0.021	0.021	0.188	0.002	0.031	−0.136	−0.194	−0.029	0.264
18	0.020	0.020	0.141	−0.002	0.031	−0.111	−0.278	−0.033	0.211
19	0.019	0.019	0.094	−0.007	0.031	−0.081	−0.452	−0.038	0.151
20									

Appendix 2

A general discrete-time model can be written as follows:

$$\begin{aligned} \max_{C_t, R_t, M_t} \sum_{t=1}^{T} U(C_t, E(M_t, R_t)) \cdot (1+r)^{-t} \text{s.t.} \\ K_{t+1} - K_t = F(K_t, R_t) - C_t - \delta \cdot K_t \\ S_{t+1} - S_t = g(S_t) - R_t \\ Z_{t+1} - Z_t = h(Z_t) - M_t \end{aligned} \tag{7.1}$$

Where $g(S_t)$ is the function of natural renewal of the stock of productive natural resources. The Hamiltonian and the first-order conditions are:

$$\begin{aligned} H_t = {} & U(C_t, E(M_t, R_t)) + \gamma_{t+1}^K \cdot (F(K_t, R_t) - C_t - \delta \cdot K_t) + \gamma_{t+1}^S \cdot (g(S_t) - R_t) \\ & + \gamma_{t+1}^Z \cdot (h(Z_t) - M_t) \end{aligned} \tag{7.2}$$

$$\begin{aligned} & \frac{\partial H_t}{\partial C_t} = 0 \Rightarrow U_{C_t} - \gamma_{t+1}^K = 0 \\ & \frac{\partial H_t}{\partial R_t} = 0 \Rightarrow U_{E_t} E_{R_t} + \gamma_{t+1}^K \cdot F_{R_t} - \gamma_{t+1}^S = 0 \\ & \frac{\partial H_t}{\partial M_t} = 0 \Rightarrow U_{E_t} E_{M_t} - \gamma_{t+1}^Z = 0 \end{aligned} \tag{7.3}$$

$$\begin{aligned} & \gamma_{t+1}^K - \gamma_t^K = r \cdot \gamma_t^K - \frac{\partial H_t}{\partial K_t} \Rightarrow \gamma_t^K = \frac{1 + F_{K_t} - \delta}{1+r} \cdot \gamma_{t+1}^K \\ & \gamma_{t+1}^S - \gamma_t^S = r \cdot \gamma_t^S - \frac{\partial H_t}{\partial S_t} \Rightarrow \gamma_t^S = \frac{1 + g(S_t)}{1+r} \cdot \gamma_{t+1}^S \\ & \gamma_{t+1}^Z - \gamma_t^Z = r \cdot \gamma_t^Z - \frac{\partial H_t}{\partial Z_t} \Rightarrow \gamma_t^Z = \frac{1 + h(Z_t)}{1+r} \cdot \gamma_{t+1}^Z \end{aligned} \tag{7.4}$$

The first-order conditions can be reformulated as follows:

$$\begin{aligned}\frac{U_{C_{t+1}}}{U_{C_t}} &= \frac{1+r}{1+F_{K_t}-\delta}\\ \frac{U_{C_{t+1}}F_{R_{t+1}}+U_{E_{t+1}}E_{R_{t+1}}}{U_{C_t}F_{R_t}+U_{E_t}E_{R_t}} &= \frac{1+r}{1+g(S_t)}\\ \frac{U_{E_{t+1}}E_{M_{t+1}}}{U_{E_t}E_{M_t}} &= \frac{1+r}{1+h(Z_t)}\end{aligned} \tag{7.5}$$

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